Universidade Federal de Uberlândia
Faculdade de Engenharia Química
Programa de Pós-Graduação

Uma Contribuição ao Estudo de Controladores Preditivos Robustos

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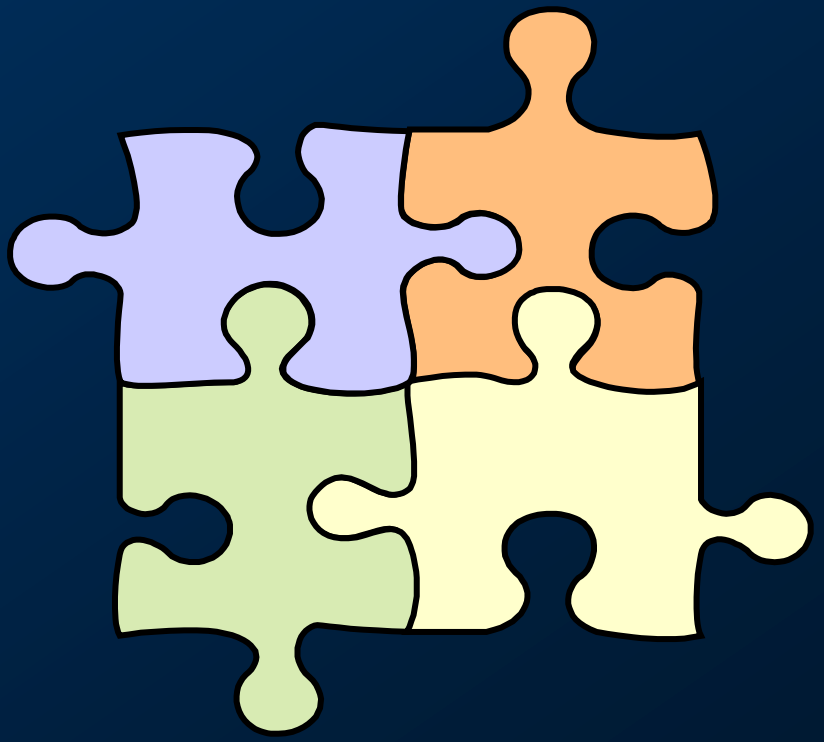


INTRODUÇÃO

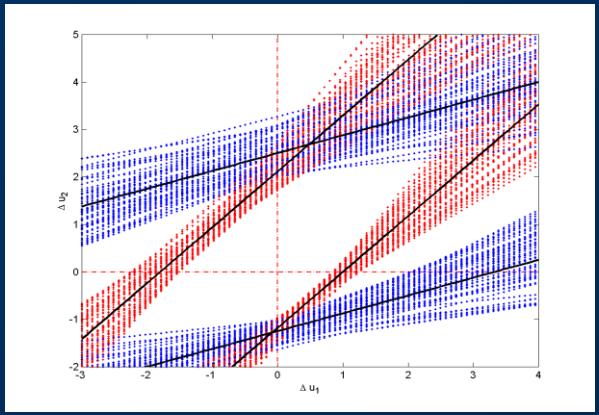
Introdução

- MPC
  - Conceito: modelo, ruído e função custo;
  - Aplicação Industrial;
- RMPC
  - Definição e Formas;

- Justificativas
  - Controle com Restrição;
  - Problemas não lineares e Incertezas;
  - Obstáculos a RMPC;

Objetivos Específicos

- O objetivo deste trabalho é propor algoritmos de controle preditivo robusto por construção aplicados a processos e que mantenham a estabilidade robusta e o desempenho do sistema em patamares aceitáveis.



Avaliação dos RMPC propostos:

- Simulação em modelos representativos de processos reais;
- Implementação em uma planta piloto – NL.

Treinamento em Pesquisa
Desenvolvimento de MPC e RMPC
Aproximar a teoria da prática industrial

Linha do Tempo

Períodos do Controle Automático:

- **Pré-história;**
- **Primitivo;**
- **Clássico;**
- **Moderno.**

Preocupação de árabe e gregos

Revolução Industrial

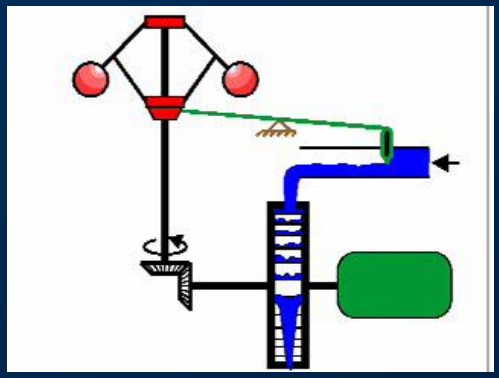
Eras: Espacial e Computadores

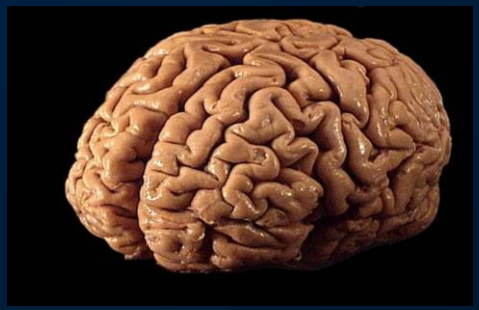
Guerras Mundiais

Comunicação em Massa

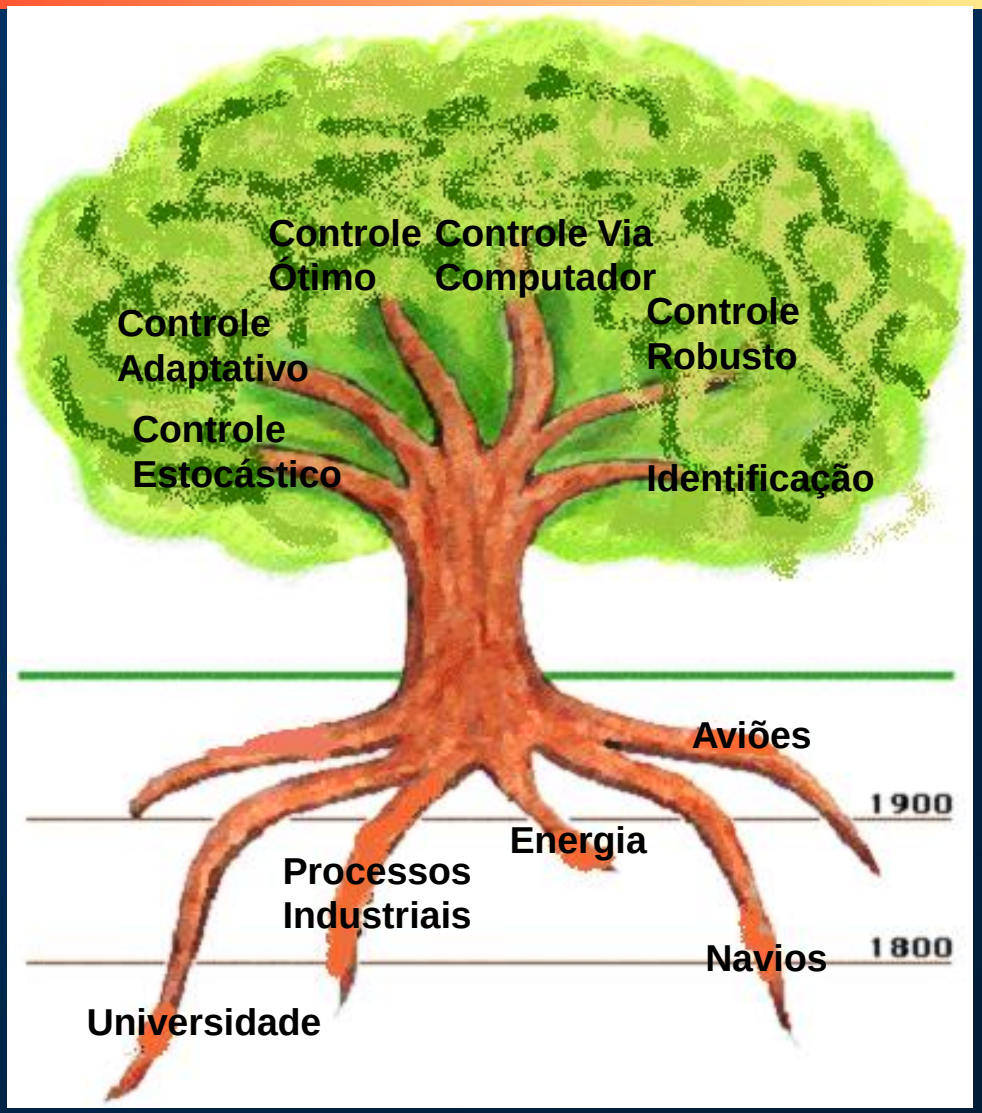
Fatos Relevantes

- Templo de Hero;
- James Watt;
- 1868: Maxwell → Wischenegradsky
- 1922: Minorsky;
- 1920-1930: Laboratórios Bell
- 1932: Nyquist;
- 1934: Hazen, Black;
- Guerras mundiais e os Sistemas de Controle;
- 1940-1950: Evans, Ziegler, Nichols, Bode;
- 1950: Realimentação Ótima (ISE);
- 1960: Computadores/Controle Moderno
- 1970 e posterior: Youla;
  Zames;
  Doyle
- 1990 até a atualidade: Controle Robusto
  Sistemas Especialistas
  Inteligência Artificial...



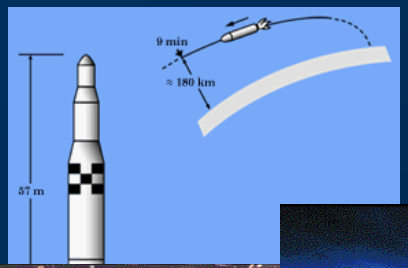


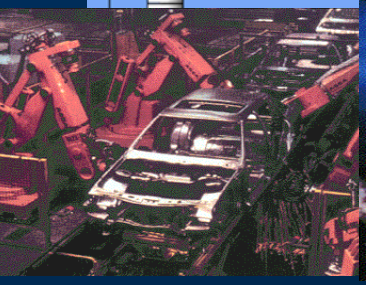
Evolução e Integração das Teorias

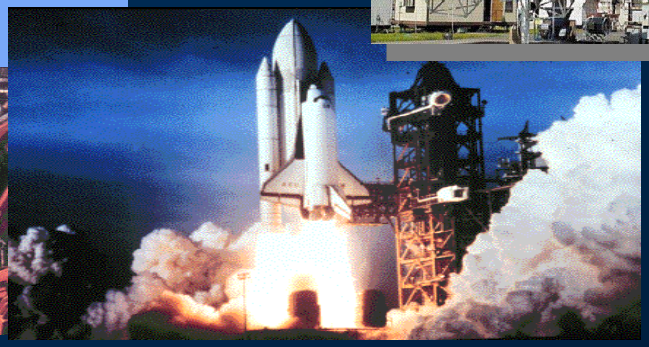


Controle Preditivo

- MPC: Extensivamente Estudado
  - Sólida Fundamentação Teórica







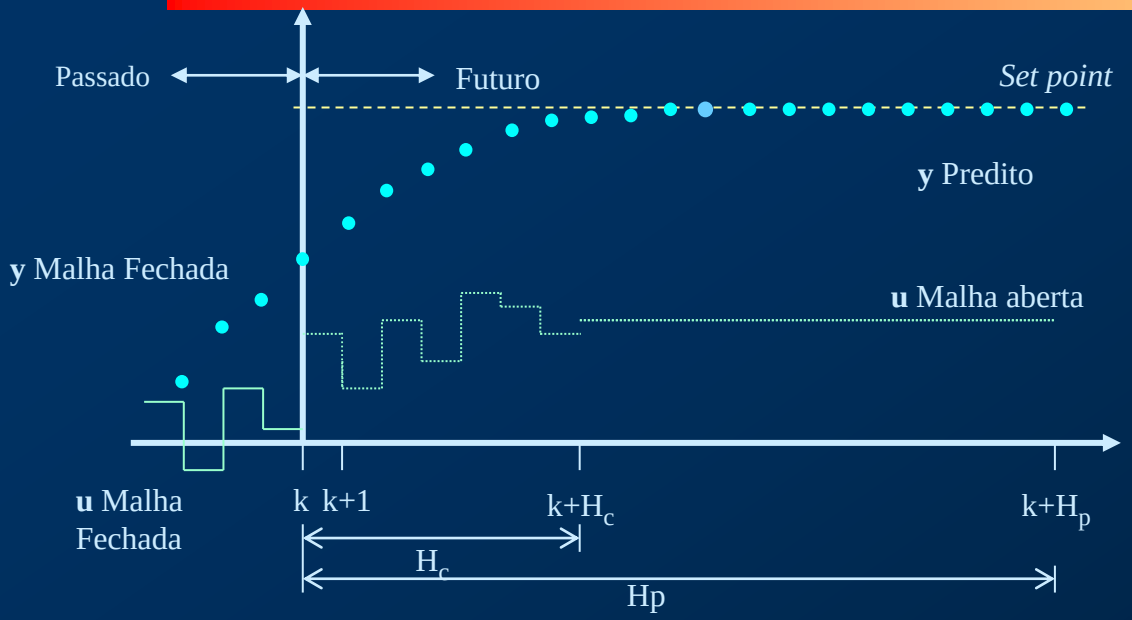
Origens e principais idéias

- Zadeh e Whalen, 1962;
- Adaptativo;

Etapas:

- Onde?
- Para onde?
- Onde queremos?
- Qual a melhor forma?

MPC - Formulações



Objetivo: $\min_{U_k} J(U_k; H_c, H_p,)$

Restrições: $\psi(U_k, Y_k) \leq 0$

- Formulações: Linear, Não linear, Modelos E/S, Espaço de Estados, Redes Neuronais, Série de Volterra, etc
- Diferentes funções objetivo
- Restrições de Estabilidade

- Pontos Fortes:
  - Desenvolvimento de Teoria, Simples Objetivos Aplicáveis a todos POs
- Pontos Fracos
  - Sintonia em geral por tentativa e erro em cada PO!
  - Necessidade ajuste para Estabilidade/Desempenho

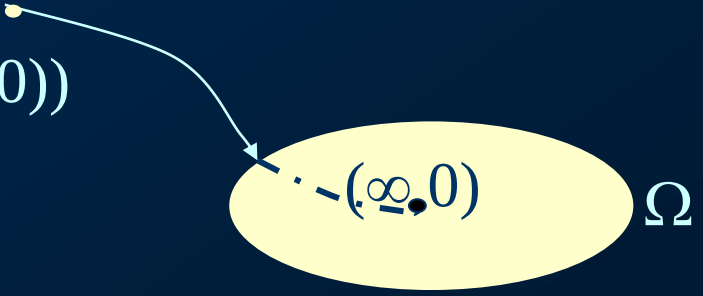
Detalhes sobre Estabilidade para MPC

Horizonte de Saída Infinito

Matriz de Pesos Terminal

Restrição terminal de igualdade $x(k+H_p)=0$

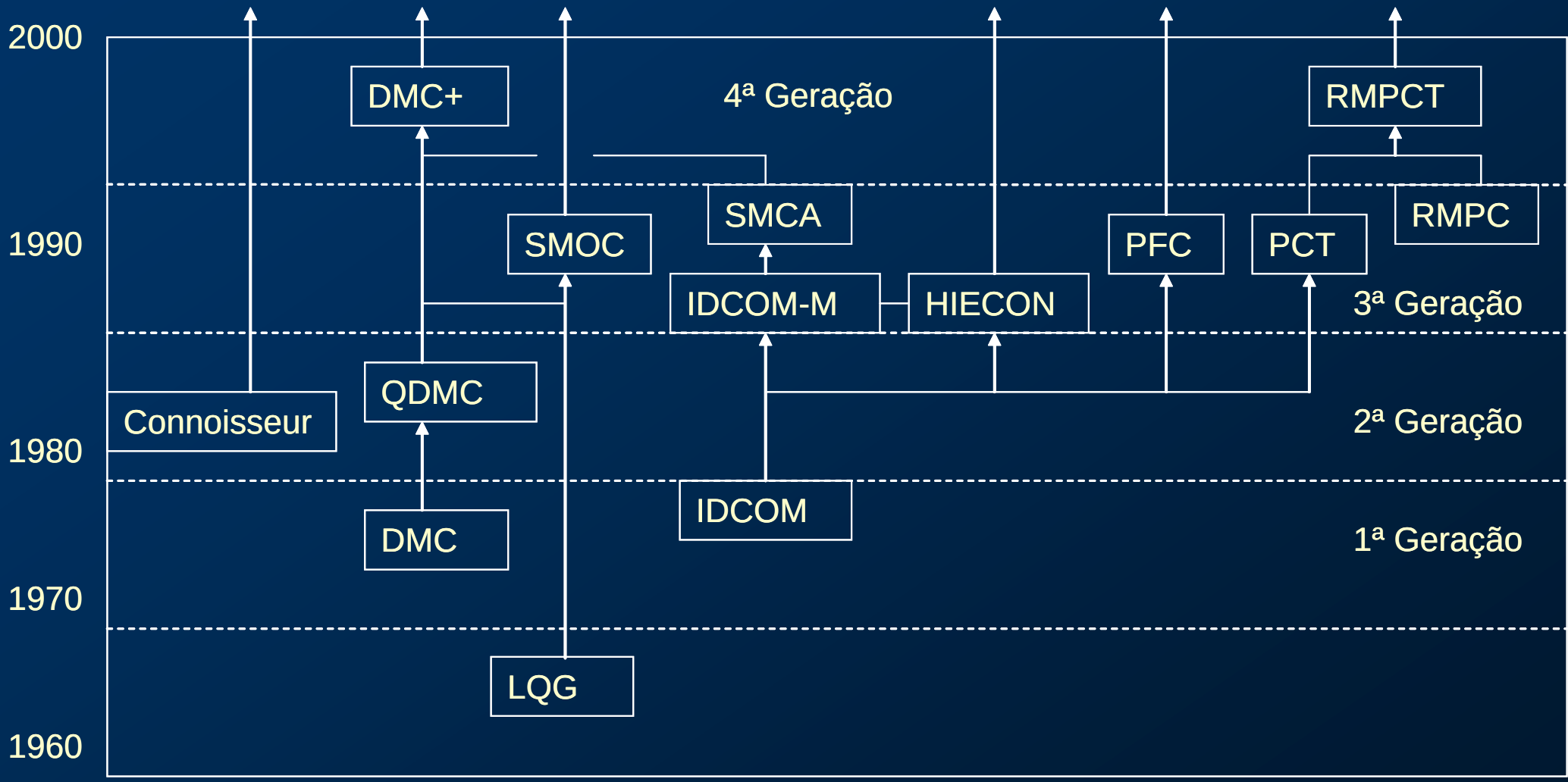
Horizonte variável modo dual $x(k+H_p) \in \Omega$



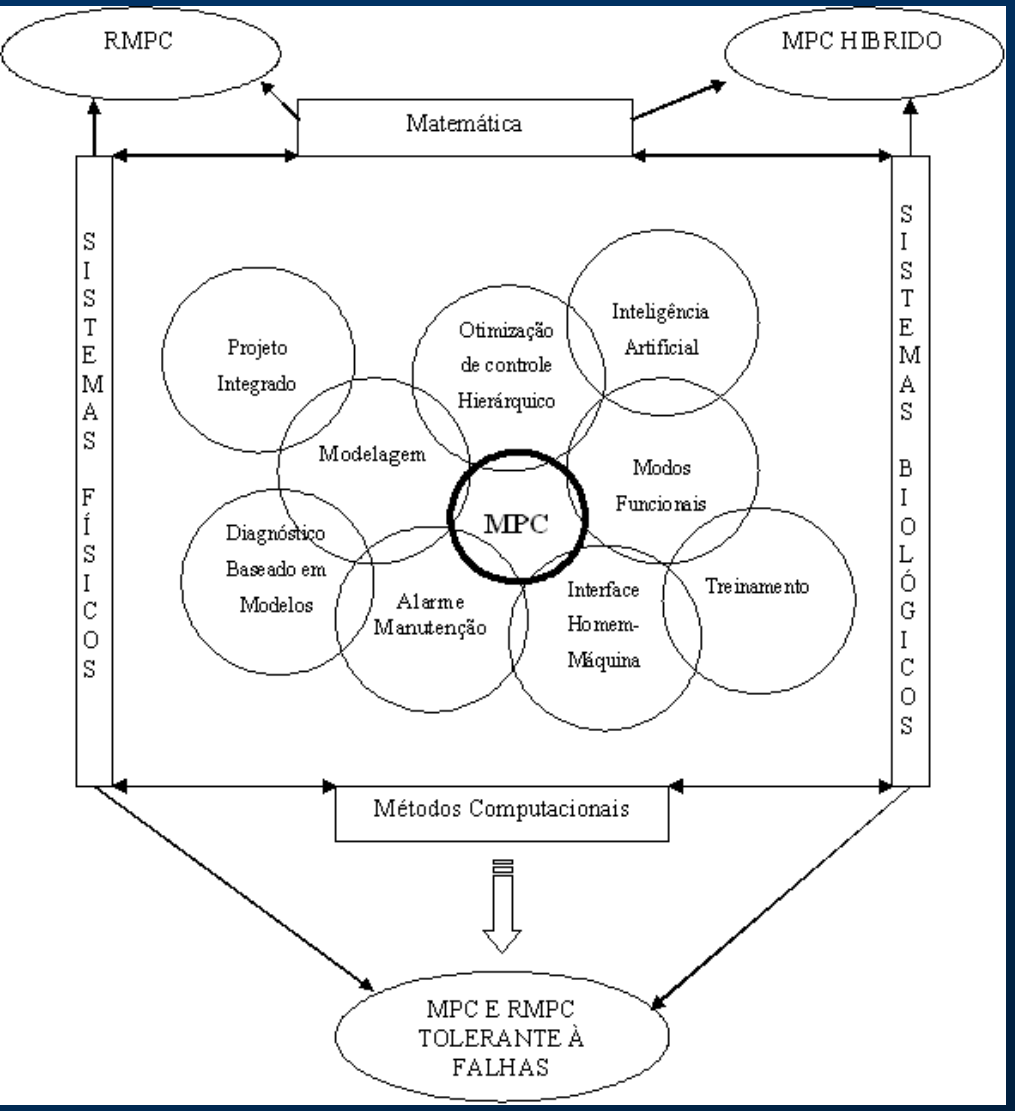
Restrição de contração $\|x(k+H_p)\|^2 \leq \alpha \|x(k)\|^2 ; \alpha \in (0,1)$

Penalidade terminal $J_q = J + \Xi\left[x(k+H_p)\right]$

Controle Preditivo



Fonte: Adaptado Qin e Bagdwell (2003)



Desenvolvimentos em Controle Preditivo

Algoritmo Base: IHMPC

$$J_k = \min_{\mathbf{u}} \sum_{i=0}^{\infty} \mathbf{y}_{k+i}^T \mathbf{Q}\mathbf{y}_{k+i} + \mathbf{u}_{k+i}^T \mathbf{R}\mathbf{u}_{k+i} + \Delta\mathbf{u}_{k+i}^T \mathbf{S}\Delta\mathbf{u}_{k+i}$$

$$J_k = \min_{\Xi} \Xi^T \mathbf{H}\Xi + 2\Xi^T(\mathbf{G}\mathbf{z}_k - \mathbf{F}\mathbf{u}_{k-1})$$

Onde:

$$\mathbf{H} = \begin{bmatrix} \mathbf{B}^T\overline{\mathbf{Q}}\mathbf{B} + \mathbf{R} + 2\mathbf{S} & \mathbf{B}^T\mathbf{A}^T\overline{\mathbf{Q}}\mathbf{B} - \mathbf{S} & \Lambda & \mathbf{B}^T\mathbf{A}^{T^{N_m-1}}\overline{\mathbf{Q}}\mathbf{B} \\ \mathbf{B}^T\overline{\mathbf{Q}}\mathbf{A}\mathbf{B} - \mathbf{S} & \mathbf{B}^T\overline{\mathbf{Q}}\mathbf{B} + \mathbf{R} + 2\mathbf{S} & \Lambda & \mathbf{B}^T\mathbf{A}^{T^{N_m-2}}\overline{\mathbf{Q}}\mathbf{B} \\ M & M & O & M \\ \mathbf{B}^T\overline{\mathbf{Q}}\mathbf{A}^{N_m-1}\mathbf{B} & \mathbf{B}^T\overline{\mathbf{Q}}\mathbf{A}^{N_m-2}\mathbf{B} & \Lambda & \mathbf{B}^T\overline{\mathbf{Q}}\mathbf{B} + \mathbf{R} + 2\mathbf{S} \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{B}^T\overline{\mathbf{Q}}\mathbf{A} \\ \mathbf{B}^T\overline{\mathbf{Q}}\mathbf{A}^2 \\ M \\ \mathbf{B}^T\overline{\mathbf{Q}}\mathbf{A}^{N_m} \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} \mathbf{S} \\ \mathbf{0} \\ M \\ \mathbf{0} \end{bmatrix}$$

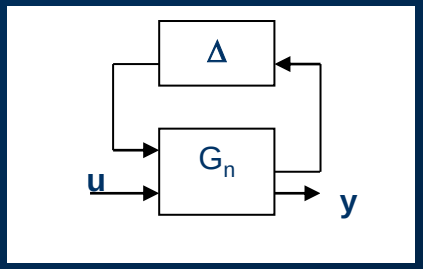
$$\mathbf{Q} = \mathbf{Q}^T \geq \mathbf{0}$$
$$\mathbf{R} = \mathbf{R}^T > \mathbf{0}$$
$$\mathbf{S} = \mathbf{S}^T \geq \mathbf{0}$$
$$\Delta\mathbf{u}_{k+i} = \mathbf{u}_{k+i} - \mathbf{u}_{k+i-1}$$

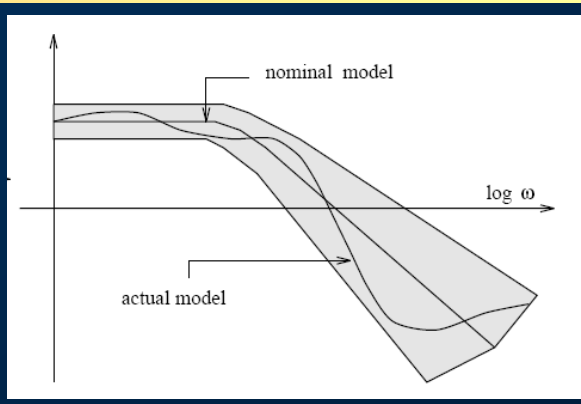
Muske e Rawlings (1993)

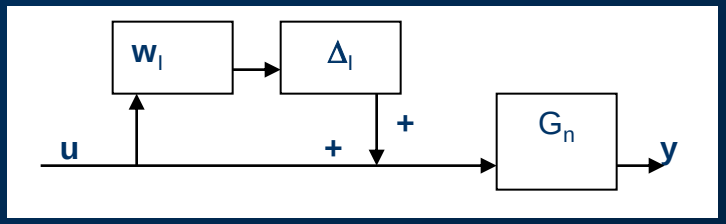
Incerteza e Robustez

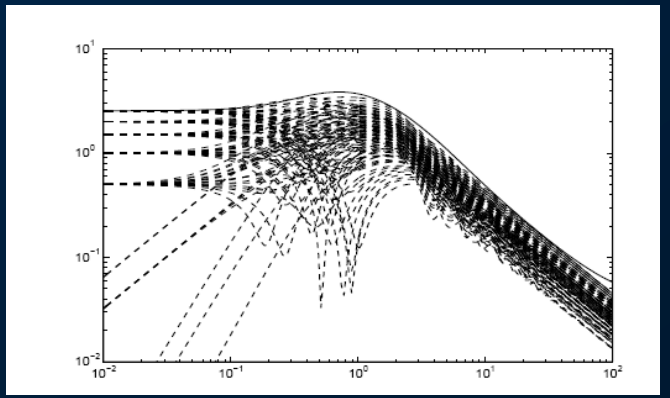
- Incerteza: origens prováveis.
  - Paramétrica;
  - Dinâmica negligenciada;
  - Distribuída;
  - Estocástica;
  - Politópica

$$G_S = \vartheta G_n(s);\ \vartheta_{mín} \le \vartheta \le \vartheta_{máx}$$









$$\mathbf{x}(t+1) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t)$$

$$\left[\mathbf{A}(t) \quad \mathbf{B}(t)\right] \in \Pi$$

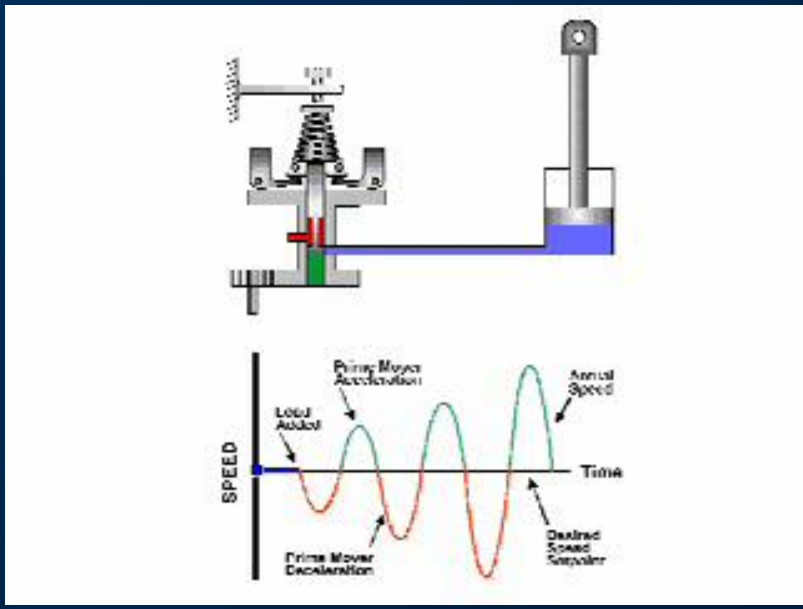
$$\Pi = \mathrm{co}\left\{\left[\mathbf{A}_1(t) \quad \mathbf{B}_1(t)\right] \ \Lambda \ \left[\mathbf{A}_M(t) \quad \mathbf{B}_M(t)\right]\right\}$$

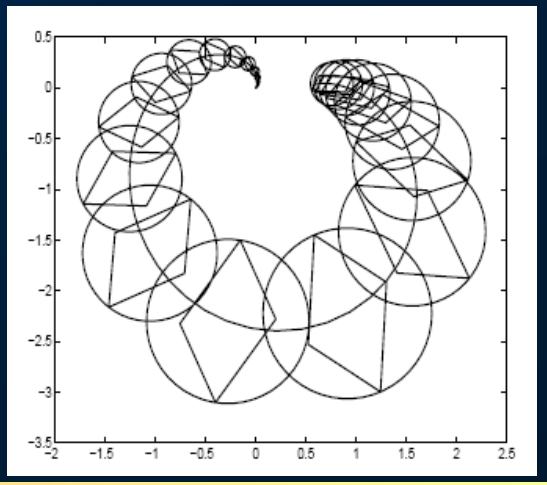
Justificativas:

- Estabilidade e Desempenho
- Modelos matemáticos e o RMPC;

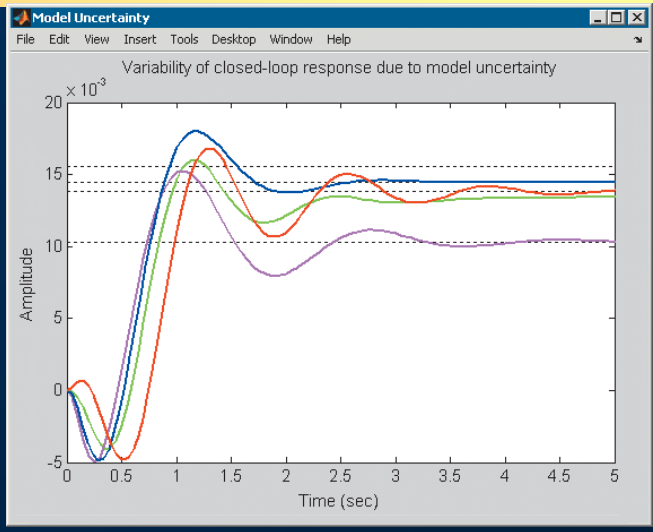
$$\min_{\mathbf{u}\in\mathbf{U}} \max_{\theta\in\Theta} J(\mathbf{u},\theta)$$

- RMPC: controladores conservativos;

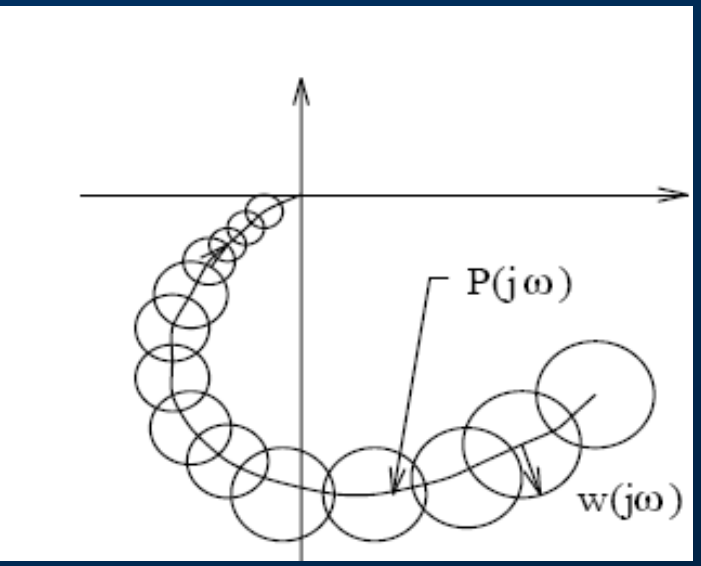




- Análise
  - MPC padrão com modelo nominal;
  - Por construção.



- Síntese
  - Robustez inerente;
    - Mín-máx.



- Estabilidade
  - Escolher parâmetros para MPC padrão;
  - Modificar problema adicionando restrição.

Algoritmo Base:RMPC_MMTV

$$\min_{\Xi\in\Xi^m}\max_{\Theta\in\Theta^m}\left[J_k(\Xi_k,\theta_k)\overset{\Delta}{=}\mathbf{X}_{k+1}{}^T\boldsymbol{\mu}\mathbf{X}_{k+1}+\Xi_k{}^T\Lambda\Xi_k\right]$$

Sujeito a:

$$\mathbf{X}_{k+1}=\overline{\mathbf{A}}\mathbf{z}_k+\overline{\mathbf{B}}(\theta_k)\Xi_k$$

Em que:

$$\mathbf{X}_{k+1}=\begin{bmatrix}\mathbf{x}_{k+1} & \Lambda & \mathbf{x}_{k+Nm}\end{bmatrix}^T$$

$$\Xi_{k+1}=\begin{bmatrix}\mathbf{u}_k & \Lambda & \mathbf{u}_{k+Nm-1}\end{bmatrix}^T$$

$$\theta_k=\begin{bmatrix}\delta_1 & \Lambda & \delta_{\dim(\mathbf{u})}\end{bmatrix}^T$$

$$\overline{\mathbf{Q}}=\mathbf{Q}+\mathbf{A}^T\overline{\mathbf{Q}}\mathbf{A}$$

$$\mathbf{U}^{N_m}=\underset{N_m}{\mathbf{U}_{123}}$$

$$\mathbf{X}^{N_m}=\underset{N_m}{\mathbf{X}_{123}}$$

$$\Lambda=\operatorname{diag}\left[\underset{N_m}{\mathbf{R}_{123}}\right]$$

$$\boldsymbol{\mu}=\operatorname{diag}\left[\underset{N_m-1}{\mathbf{Q}_{123}}\ \overline{\mathbf{Q}}\right]$$

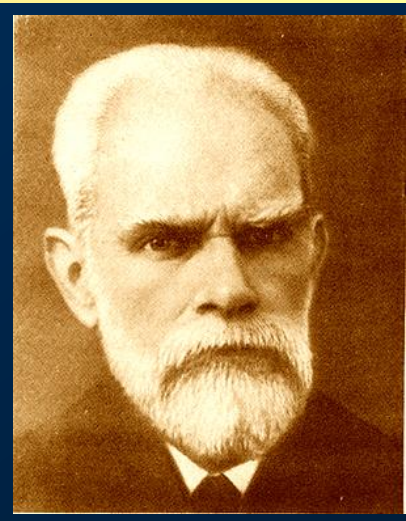
$$\overline{\mathbf{A}}=\begin{bmatrix}\mathbf{A}\\ \mathbf{M}\\ \mathbf{A}^{N_m}\end{bmatrix}$$

$$\overline{\mathbf{B}}(\theta_k)=\begin{bmatrix}\mathbf{B}(\theta_k) & \mathbf{0} & \Lambda & \mathbf{0}\\ \mathbf{AB}(\theta_k) & \mathbf{B}(\theta_{k+1}) & \mathrm{O} & \mathrm{M}\\ \mathrm{M} & \mathrm{O} & \mathrm{O} & \mathbf{0}\\ \mathbf{A}^{N_m-1}\mathbf{B}(\theta_k) & \mathbf{A}^{N_m-1}\mathbf{B}(\theta_{k+1}) & \Lambda & \mathbf{B}(\theta_{k+N_m})\end{bmatrix}$$

$$\mathbf{B}(\theta_k)=\mathbf{B}\begin{bmatrix}1+\delta^1 & 0 & \Lambda & 0\\ 0 & 1+\delta^2 & \mathrm{M} & \mathrm{M}\\ \mathrm{M} & \Lambda & \mathrm{O} & \mathrm{M}\\ 0 & \Lambda & \Lambda & 1+\delta^{\dim(\mathbf{u})}\end{bmatrix}$$

Lee e Cooley (2000)

Inequações Matriciais Lineares - LMI



- Origem: Lyapunov;
- Aplicações: ....
- Vantagens:
  - Tempo polinominal;
  - Grau de Ótimo;
  - Critério de Parada;
  - Solução eficiente: Método utilizado.
- LMI e o RMPC.

$$\begin{bmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{bmatrix} \geq 0 \iff \begin{matrix} R(x) > 0 \\ Q(x) - S(x)R(x)^{-1}S(x)^T \geq 0 \end{matrix}$$

Algoritmo base: RMPC_PE

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{B}_d\mathbf{d}(k)$$
$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k)$$

$$\min_{\Gamma, U_{lmi}, T} \quad \Gamma$$

Sujeito a :

$$\begin{pmatrix} \Gamma - \sum_{i=1}^{s} \tau_i & \left(\mathbf{C}_{lmi}(\mathbf{A}_{lmi}\mathbf{x}_{k|k} + \mathbf{B}_{lmi}\mathbf{U}_{lmi})\right)^T & \mathbf{U}_{lmi} & 0 \\ \mathbf{C}_{lmi}(\mathbf{A}_{lmi}\mathbf{x}_{k|k} + \mathbf{B}_{lmi}\mathbf{U}_{lmi}) & \mathbf{Q}_{lmi}^{-1} & 0 & \mathbf{C}_{lmi}\mathbf{B}_{d_{lmi}} \\ \mathbf{U}_{lmi} & 0 & \mathbf{R}_{lmi}^{-1} & 0 \\ 0 & (\mathbf{C}_{lmi}\mathbf{B}_{d_{lmi}})^T & 0 & \mathbf{T} \end{pmatrix} \geq 0$$

$$\boldsymbol{E}_{\mathbf{u}}\mathbf{U}_{lmi} \leq \boldsymbol{F}_{\mathbf{u}}$$

$$\boldsymbol{E}_x(\mathbf{A}_{lmi}\mathbf{x}_{k|k} + \mathbf{B}_{lmi}\mathbf{U}_{lmi}) + \gamma \leq \boldsymbol{F}_x$$

$$\begin{pmatrix} 1-\tau & (\mathbf{A}\mathbf{z}_{k|k} + \mathbf{B}\mathbf{u}_{k|k})^T & \mathbf{0} \\ (\mathbf{A}\mathbf{z}_{k|k} + \mathbf{B}\mathbf{u}_{k|k}) & \mathbf{P}^{-1} & \mathbf{B}_d \\ \mathbf{0} & \mathbf{B}_d & \tau\mathbf{I} \end{pmatrix} \geq 0$$

Löfberg (2003)

Algoritmos Comparativos: Montandon (2005)

$$\min_{\upsilon(k),\ldots,\upsilon(k+N_m+N_p+2)} \quad J = \frac{1}{2}\upsilon^T \varepsilon \upsilon$$

sujeito a: $\mathbf{D}\upsilon = \mathbf{b}$

$$\mathbf{u}_{min}(k+i) \le \mathbf{u}(k+i) \le \mathbf{u}_{max}(k+i)$$

$$-\left|\Delta\mathbf{u}_{min}(k+i)\right| \le \Delta\mathbf{u}(k+i) \le \left|\Delta\mathbf{u}_{max}(k+i)\right|$$

$$(i = 0,\ldots,N_m)$$

Em que:

$$\varepsilon = \begin{bmatrix} \mathbf{R} & [\mathbf{0}] \\ [\mathbf{0}] & \mathbf{S} \end{bmatrix}$$

$$\begin{bmatrix}
\mathbf{CB} & [\mathbf{0}] & \Lambda & [\mathbf{0}] & [\mathbf{I}] & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & \Lambda & [\mathbf{0}] \\
(\mathbf{CA}+\mathbf{KC})\,\Psi\mathbf{B} & \mathbf{CB} & \Lambda & [\mathbf{0}] & [-\mathbf{I}] & [\mathbf{I}] & [\mathbf{0}] & [\mathbf{0}] & \Lambda & [\mathbf{0}] \\
(\mathbf{CA}+\mathbf{KC})\,\Phi\Psi\mathbf{B} & (\mathbf{CA}+\mathbf{KC})\,\Psi\mathbf{B} & \Lambda & [\mathbf{0}] & [\mathbf{0}] & [-\mathbf{I}] & [\mathbf{I}] & [\mathbf{0}] & \Lambda & [\mathbf{0}] \\
\mathbf{M} & \mathbf{M} & \Lambda & \mathbf{M} & \mathbf{M} & \mathbf{M} & \mathbf{M} & \mathbf{M} & \Lambda & \mathbf{M} \\
(\mathbf{CA}+\mathbf{KC})\,\Phi^{N_m-1}\Psi\mathbf{B} & (\mathbf{CA}+\mathbf{KC})\,\Phi^{N_m-2}\Psi\mathbf{B} & \Lambda & \mathbf{CB} & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & \Lambda & [\mathbf{I}] \\
(\mathbf{CA}+\mathbf{KC})\,\Phi^{N_m}\Psi\mathbf{B} & (\mathbf{CA}+\mathbf{KC})\,\Phi^{N_m-1}\Psi\mathbf{B} & \Lambda & (\mathbf{CA}+\mathbf{KC})\,\Phi\mathbf{B} & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & \Lambda & [\mathbf{I}] \\
(\mathbf{CA}+\mathbf{KC})\,\Phi^{N_m+1}\Psi\mathbf{B} & (\mathbf{CA}+\mathbf{KC})\,\Phi^{N_m}\Psi\mathbf{B} & \Lambda & (\mathbf{CA}+\mathbf{KC})\,\Phi\Psi\mathbf{B} & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & \Lambda & [\mathbf{I}] \\
\mathbf{M} & \mathbf{M} & \Lambda & \mathbf{M} & \mathbf{M} & \mathbf{M} & \mathbf{M} & \mathbf{M} & \Lambda & \mathbf{M} \\
(\mathbf{CA}+\mathbf{KC})\,\Phi^{N_p-1}\Psi\mathbf{B} & (\mathbf{CA}+\mathbf{KC})\,\Phi^{N_p-2}\Psi\mathbf{B} & \Lambda & (\mathbf{CA}+\mathbf{KC})\,\Phi^{N_p-N_m}\Psi\mathbf{B} & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & \Lambda & [\mathbf{I}]
\end{bmatrix}
\begin{bmatrix}
\Delta\mathbf{u}(k) \\ \Delta\mathbf{u}(k+1) \\ \Delta\mathbf{u}(k+2) \\ \mathbf{M} \\ \Delta\mathbf{u}(k+N_m) \\ \\ \lambda(k) \\ \lambda(k+1) \\ \lambda(k+2) \\ \mathbf{M} \\ \lambda(k+N_p)
\end{bmatrix}
=
\begin{bmatrix}
\mathbf{K}\left(\mathbf{y}^{SP}(k)-\mathbf{y}(k)\right)-\mathbf{CA}\Delta\mathbf{x}(k)-\dfrac{\mathbf{y}(k)-\mathbf{y}(k-1)}{\Delta t} \\
\mathbf{K}\Delta\mathbf{y}^{SP}(k+1)-(\mathbf{CA}+\mathbf{KC})\Phi\Delta\mathbf{x}(k) \\
\mathbf{K}\Delta\mathbf{y}^{SP}(k+2)-(\mathbf{CA}+\mathbf{KC})\Phi^{2}\Delta\mathbf{x}(k) \\
\mathbf{M} \\
\mathbf{K}\Delta\mathbf{y}^{SP}(k+N_m)-(\mathbf{CA}+\mathbf{KC})\Phi^{N_m}\Delta\mathbf{x}(k) \\
\mathbf{K}\Delta\mathbf{y}^{SP}(k+N_m+1)-(\mathbf{CA}+\mathbf{KC})\Phi^{N_m+1}\Delta\mathbf{x}(k) \\
\mathbf{K}\Delta\mathbf{y}^{SP}(k+N_m+2)-(\mathbf{CA}+\mathbf{KC})\Phi^{N_m+2}\Delta\mathbf{x}(k) \\
\mathbf{M} \\
\mathbf{K}\Delta\mathbf{y}^{SP}(k+N_p)-(\mathbf{CA}+\mathbf{KC})\Phi^{N_p}\Delta\mathbf{x}(k)
\end{bmatrix}$$

DESENVOLVIMENTO

RMPC

$$\min_{\Xi\in\Xi^m}\max_{\Theta\in\Theta^m}\left[J_k(\Xi_k,\theta_k)\overset{\Delta}{=}\mathbf{X}_{k+1}{}^T\boldsymbol{\mu}\mathbf{X}_{k+1}+\Xi_k{}^T\Lambda\Xi_k\right]$$

Sujeito a:

$$\mathbf{X}_{k+1}=\overline{\mathbf{A}}\mathbf{z}_k+\overline{\mathbf{B}}(\boldsymbol{\theta}_k)\Xi_k$$

Em que:

$$\mathbf{X}_{k+1}=\left[\mathbf{x}_{k+1}\quad\Lambda\quad\mathbf{x}_{k+Nm}\right]^T$$

$$\Xi_{k+1}=\left[\mathbf{u}_k\quad\Lambda\quad\mathbf{u}_{k+Nm-1}\right]^T$$

$$\boldsymbol{\theta}_k=\left[\delta_1\quad\Lambda\quad\delta_{\dim(\mathbf{u})}\right]^T$$

$$\overline{\mathbf{Q}}=\mathbf{Q}+\mathbf{A}^T\overline{\mathbf{Q}}\mathbf{A}$$

$$\mathbf{U}^{N_m}=\underset{N_m}{\mathbf{U}_{123}}$$

$$\mathbf{X}^{N_m}=\underset{N_m}{\mathbf{X}_{123}}$$

$$\Lambda=\operatorname{diag}\left[\underset{N_m}{\mathbf{R}_{123}}\right]$$

$$\boldsymbol{\mu}=\operatorname{diag}\left[\underset{N_m-1}{\mathbf{Q}_{123}}\quad\overline{\mathbf{Q}}\right]$$

$$\overline{\mathbf{A}}=\begin{bmatrix}\mathbf{A}\\ \mathbf{M}\\ \mathbf{A}^{N_m}\end{bmatrix}$$

$$\overline{\mathbf{B}}(\Theta_k)=\begin{bmatrix}\mathbf{B}(\boldsymbol{\theta}_k) & \mathbf{0} & \Lambda & \mathbf{0}\\ \mathbf{AB}(\boldsymbol{\theta}_k) & \mathbf{B}(\boldsymbol{\theta}_k) & \mathrm{O} & \mathrm{M}\\ \mathrm{M} & \mathrm{O} & \mathrm{O} & \mathbf{0}\\ \mathbf{A}^{m-1}\mathbf{B}(\boldsymbol{\theta}_k) & \mathbf{A}^{m-1}\mathbf{B}(\boldsymbol{\theta}_k) & \Lambda & \mathbf{B}(\boldsymbol{\theta}_k)\end{bmatrix}$$

$$\mathbf{B}(\boldsymbol{\theta}_k)=\mathbf{B}\begin{bmatrix}1+\delta^1 & 0 & \Lambda & 0\\ 0 & 1+\delta^2 & \mathrm{M} & \mathrm{M}\\ \mathrm{M} & \Lambda & \mathrm{O} & \mathrm{M}\\ 0 & \Lambda & \Lambda & 1+\delta^{\dim(\mathbf{u})}\end{bmatrix}$$

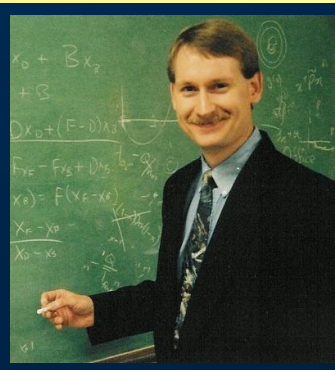
Lee e Cooley (2000)

RMPC_MMR :Restrição de Robustez

$$J_k(\Xi_k, \theta_k) \leq J_k(\Xi_k^*, \theta_k), \forall \theta_k \in \Theta$$

$$\overline{\Xi}_k^* = \Phi \Xi_{k-1}^*, \Phi = \begin{bmatrix} 0 & I & 0 & \Lambda & 0 \\ 0 & 0 & I & M & 0 \\ M & O & O & 0 & I \\ 0 & 0 & \Lambda & \Lambda & 0 \end{bmatrix}$$

Badgwell (1997)



$$J_k = \min_{\Xi} \max_{\Delta} \sum_0^{\infty} \Psi\left(x_{k+i}, u_{k+i}, \Delta u_{k+i}\right)$$

Sujeito à:

$$u_{k+j|k} \in U, \forall \Delta \in \Delta$$

$$x_{k+j|k} \in U, \forall \Delta \in \Delta$$

$$J_k(\Xi_k, \Delta_k) \leq J_k(\Xi_k^*, \Delta_k), \forall \Delta \in \Delta$$

$$J(k) = \sum_{i=0}^{\infty} \left(\mathbf{y}(k+i)^T \mathbf{Q}\mathbf{y}(k+i) + \mathbf{u}(k+i)^T \mathbf{R}\mathbf{u}(k+i) \right)$$

$$J(k) = \sum_{i=0}^{N_m - 1} \left(\mathbf{y}(k+i \mid k)^T \mathbf{Q}\mathbf{y}(k+i \mid k) + \mathbf{u}(k+i \mid k)^T \mathbf{R}\mathbf{u}(k+i \mid k) \right)$$

$$\mathbf{Y}_{lmi} = \begin{pmatrix} \mathbf{y}_{k|k}^T & \mathbf{y}_{k+1|k}^T & \cdots & \mathbf{y}_{k+N_m-1|k}^T \end{pmatrix}^T \quad \mathbf{X}_{lmi} = \begin{pmatrix} \mathbf{x}_{k|k}^T & \mathbf{x}_{k+1|k}^T & \cdots & \mathbf{x}_{k+N_m-1|k}^T \end{pmatrix}^T \quad \mathbf{U}_{lmi} = \begin{pmatrix} \mathbf{u}_{k|k}^T & \mathbf{u}_{k+1|k}^T & \cdots & \mathbf{u}_{k+N_m-1|k}^T \end{pmatrix}^T$$

$$\mathbf{Y}_{lmi} = \mathbf{C}_{lmi} \mathbf{X}_{lmi}$$

$$\mathbf{X}_{lmi} = \mathbf{A}_{lmi} \mathbf{x}_{k|k} + \mathbf{B}_{lmi} \mathbf{U}_{lmi}$$

$$\begin{bmatrix} \mathbf{A}(t) & \mathbf{B}(t) \end{bmatrix} \in \Pi$$

$$\Pi = co\left\{ \begin{bmatrix} \mathbf{A}_1(t) & \mathbf{B}_1(T) \end{bmatrix} \ \Lambda \ \begin{bmatrix} \mathbf{A}_M(t) & \mathbf{B}_M(T) \end{bmatrix} \right\}$$

$$\mathbf{A}_{lmi} = \begin{pmatrix} \mathbf{I} \\ \mathbf{A} \\ \mathbf{A}^2 \\ \mathrm{M} \\ \mathbf{A}^{N_m-1} \end{pmatrix} \qquad \mathbf{B}_{lmi} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \Lambda & \mathbf{0} \\ \mathbf{B} & \mathbf{0} & \mathbf{0} & \Lambda & \mathbf{0} \\ \mathbf{AB} & \mathbf{B} & \mathbf{0} & \Lambda & \mathbf{0} \\ \mathrm{M} & \mathrm{O} & \mathrm{O} & \mathrm{O} & \mathrm{M} \\ \mathbf{A}^{N_m-2}\mathbf{B} & \Lambda & \mathbf{AB} & \mathbf{B} & \mathbf{0} \end{bmatrix}$$

$$\mathbf{C}_{lmi} = \begin{pmatrix} \mathbf{C} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathrm{O} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C} \end{pmatrix} \qquad \mathbf{R}_{lmi} = \begin{pmatrix} \mathbf{R} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathrm{O} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R} \end{pmatrix} \qquad \mathbf{Q}_{lmi} = \begin{pmatrix} \mathbf{Q} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathrm{O} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Q} \end{pmatrix}$$

$$\mathbf{E}_{u_{lmi}} = \begin{pmatrix} \mathbf{E}_u & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathrm{O} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E}_u \end{pmatrix}, \mathbf{F}_{u_{lmi}} = \begin{pmatrix} \mathbf{f}_u^T & \mathbf{f}_u^T & \Lambda & \mathbf{f}_u^T \end{pmatrix}^T \qquad \mathbf{E}_{x_{lmi}} = \begin{pmatrix} \mathbf{E}_x & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathrm{O} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E}_x \end{pmatrix}, \mathbf{F}_{x_{lmi}} = \begin{pmatrix} \mathbf{f}_x^T & \mathbf{f}_x^T & \Lambda & \mathbf{f}_x^T \end{pmatrix}^T$$

$$\min_{U} \; \mathbf{Y}_{lmi}{}^{T}\mathbf{Q}_{lmi}\mathbf{Y}_{lmi} + \mathbf{U}_{lmi}{}^{T}\mathbf{R}_{lmi}\mathbf{U}_{lmi}$$

sujeito :

$$\boldsymbol{E}_{\mathbf{u}}\mathbf{U}_{lmi} \le \boldsymbol{F}_{\mathbf{u}}$$

$$\boldsymbol{E}_{\mathrm{x}}\mathbf{U}_{lmi} \le \boldsymbol{F}_{\mathrm{x}}$$

$$\min_{\chi,\mathbf{U}_{lmi}} \; \chi$$

sujeito :

$$\mathbf{Y}_{lmi}{}^{T}\mathbf{Q}_{lmi}\mathbf{Y}_{lmi} + \mathbf{U}_{lmi}{}^{T}\mathbf{R}_{lmi}\mathbf{U}_{lmi} \le \chi$$

$$\begin{pmatrix} \boldsymbol{E}_{\mathbf{u}} \\ \boldsymbol{E}_{\mathrm{x}}\mathbf{B}_{lmi} \end{pmatrix}\mathbf{U} \le \begin{pmatrix} \boldsymbol{F}_{\mathbf{u}} \\ \boldsymbol{F}_{\mathrm{x}} - \boldsymbol{E}_{\mathrm{x}}\mathbf{A}_{lmi}\mathbf{x}_{k|k} \end{pmatrix}$$

$$\min_{\chi,\mathbf{U}_{lmi}} \; \chi$$

sujeito :

$$\begin{pmatrix} \chi & (\boldsymbol{C}(\mathbf{A}_{i,lmi}\mathbf{x}_{k|k} + \mathbf{B}_{i,lmi}\mathbf{U}_{lmi}))^{T} & \mathbf{U}_{lmi}{}^{T} \\ (\mathbf{C}_{lmi}(\mathbf{A}_{i,lmi}\mathbf{x}_{k|k} + \mathbf{B}_{i,lmi}\mathbf{U}_{lmi})) & \mathbf{Q}_{lmi}{}^{-1} & \mathbf{0} \\ \mathbf{U} & \mathbf{0} & \mathbf{R}_{lmi}{}^{-1} \end{pmatrix} \ge 0$$

$$\begin{pmatrix} \boldsymbol{E}_{\mathbf{u}} \\ \boldsymbol{E}_{\mathrm{x}}\mathbf{B}_{i,lmi} \end{pmatrix}\mathbf{U}_{lmi} \le \begin{pmatrix} \boldsymbol{F}_{\mathbf{u}} \\ \boldsymbol{F}_{\mathrm{x}} - \boldsymbol{E}_{\mathrm{x}}\mathbf{A}_{i,lmi}\mathbf{x}_{k|k} \end{pmatrix}$$

Restrição de Robustez

$$\begin{pmatrix} 1 & (\mathbf{A}_{i}\mathbf{z}_{k|k} + \mathbf{B}_{i}\mathbf{u}_{k|k})^{T} \\ (\mathbf{A}_{i}\mathbf{z}_{k|k} + \mathbf{B}_{i}\mathbf{u}_{k|k}) & \mathbf{P}^{-1} \end{pmatrix} \ge 0$$

RESULTADOS

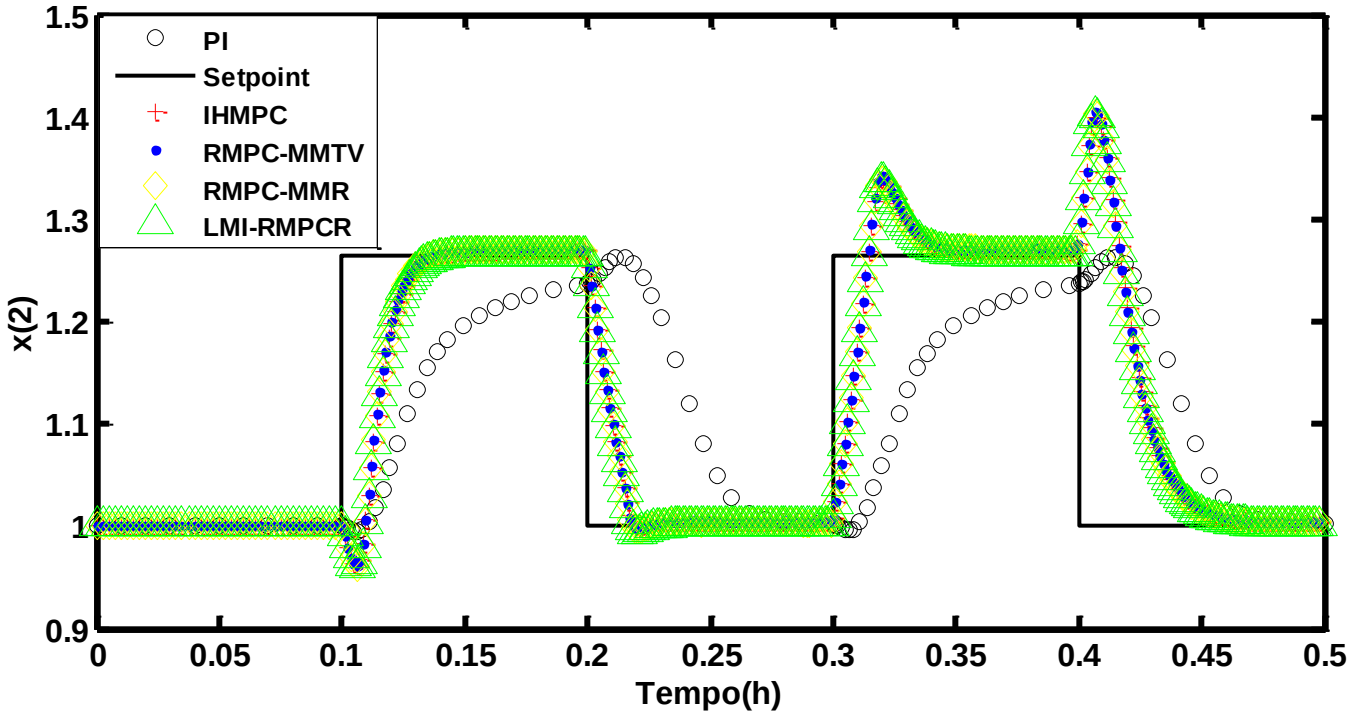
SIMULAÇÃO

Simulação: CSTR Isotérmico

$$\dot{x}_1 = -k_1 x_1 - k_3 x_1^2 + (x_F - x_1)u$$

$$\dot{x}_2 = k_1 x_1 - k_2 x_2 - x_2 u$$

$$A \xrightarrow{k_1} B \qquad B \xrightarrow{k_2} C \qquad 2A \xrightarrow{k_3} D$$

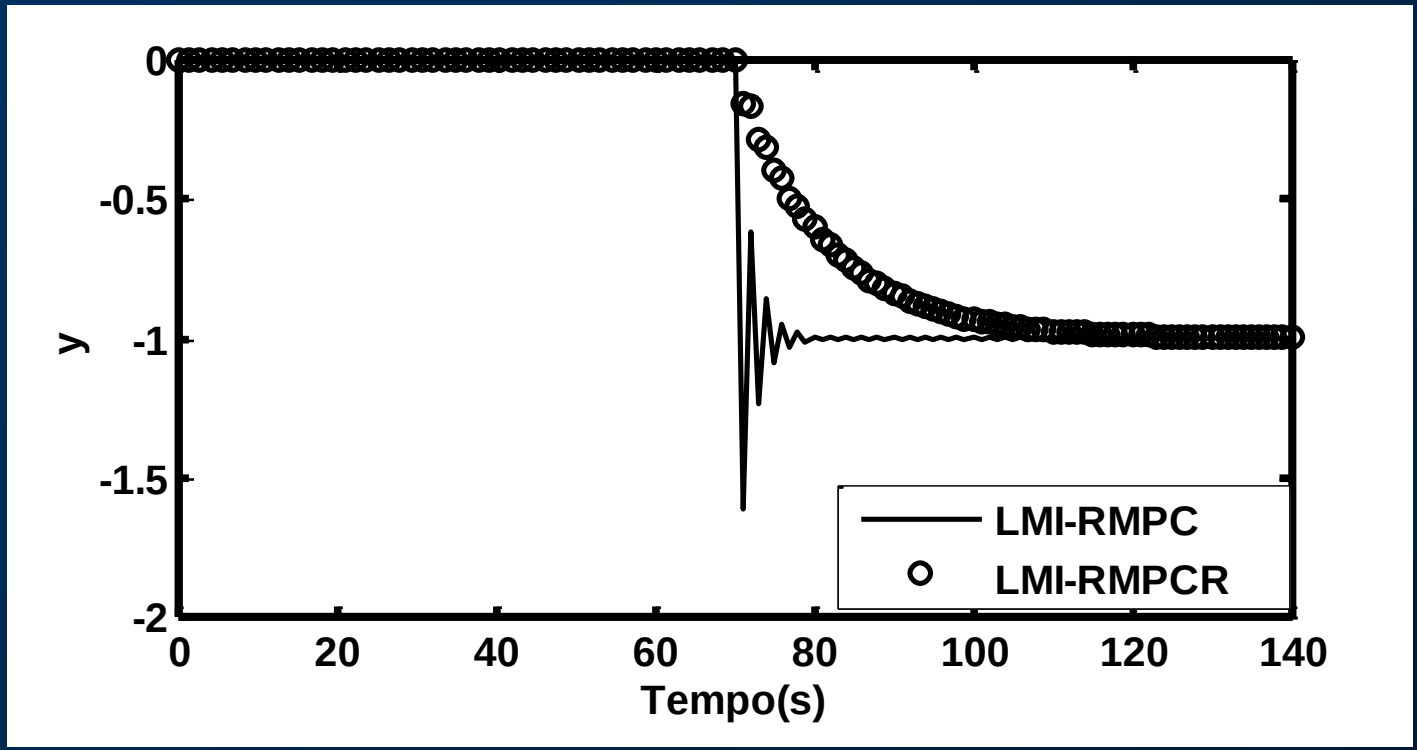


Multiplicidade de ee em **u**

Simulação: Modelo instável/Planta estável

$$G_1 = \frac{1}{z - 0{,}5}$$

$$G_2 = \frac{1}{z - 1{,}6}$$

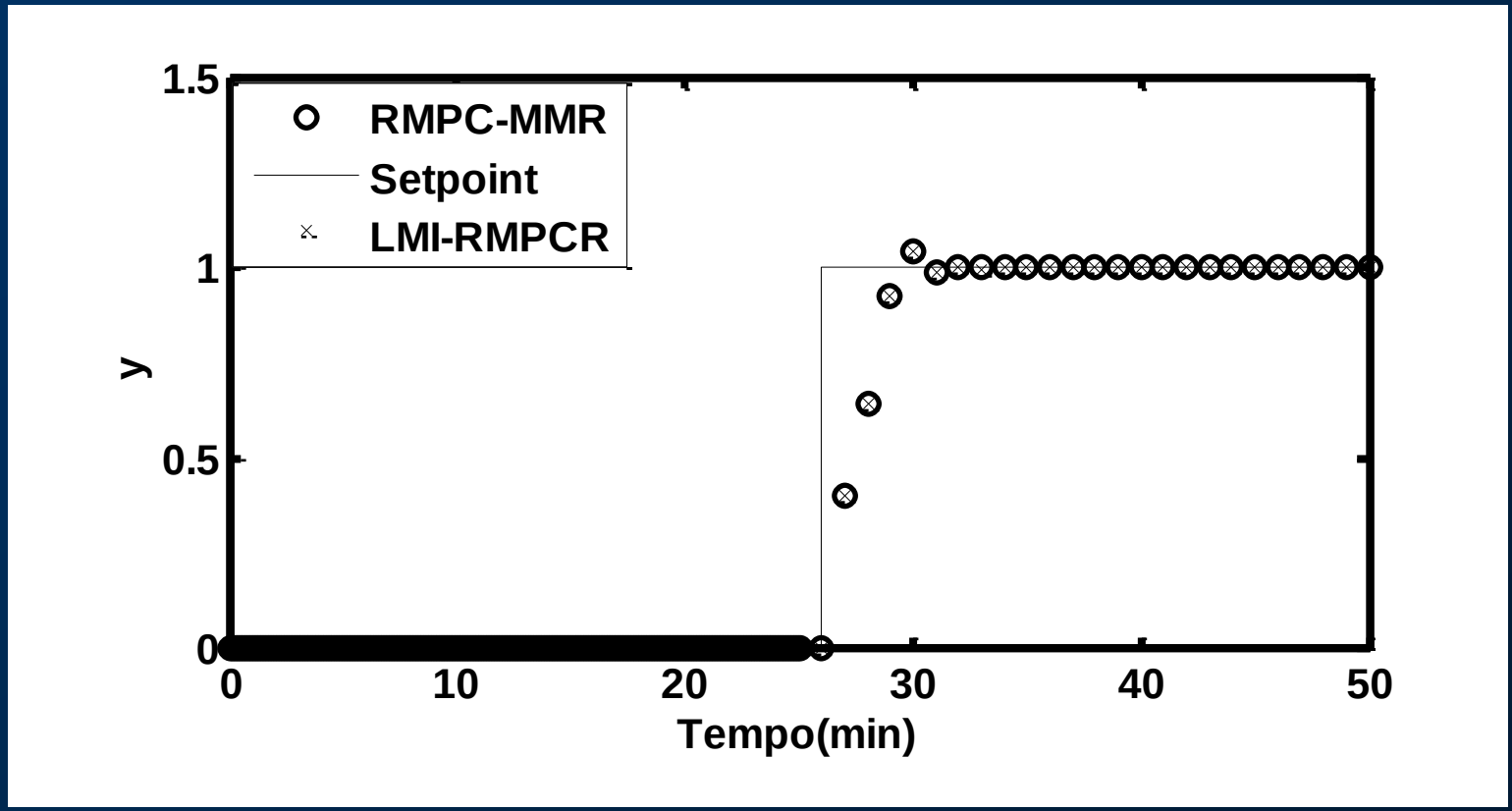


Controladores:
LMI_RMPC - Kothare
LMI_RMPCR - Desenvolvido

Simulação: Discrepância de Tempo Morto

$$G1 = \frac{3e^{-10s}}{(4s+1)^2}$$

$$G2 = \frac{1e^{-5s}}{(5s+1)^2}$$



Controladores:
RMPC_MMR
LMI_RMPCR

Simulação: Sistema Mal-Condicionado

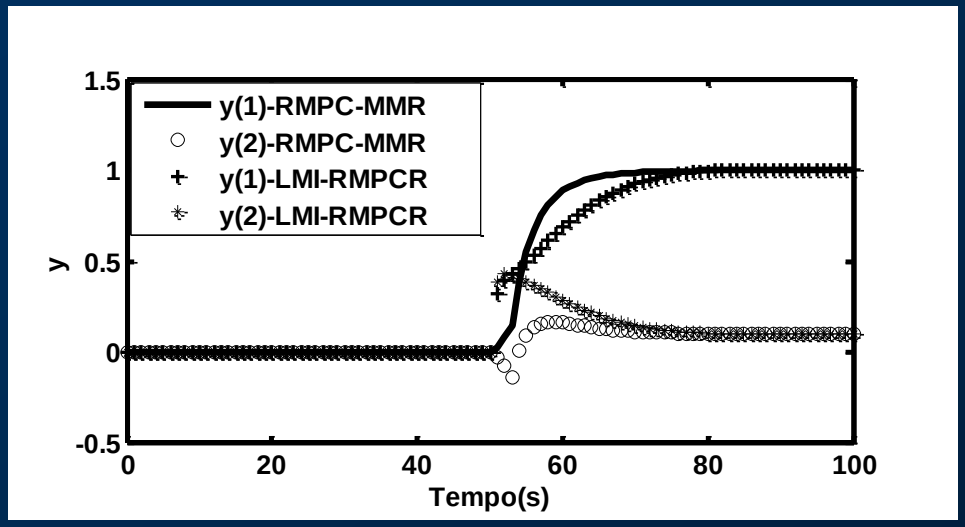
$$\mathbf{x}_{k+1} = \begin{bmatrix} 0{,}8187 & 0 \\ 0 & 0{,}8187 \end{bmatrix}\mathbf{x}_k + \begin{bmatrix} 0{,}2393 & -0{,}1563 \\ 0{,}1958 & -0{,}1985 \end{bmatrix}\mathbf{u}_k$$

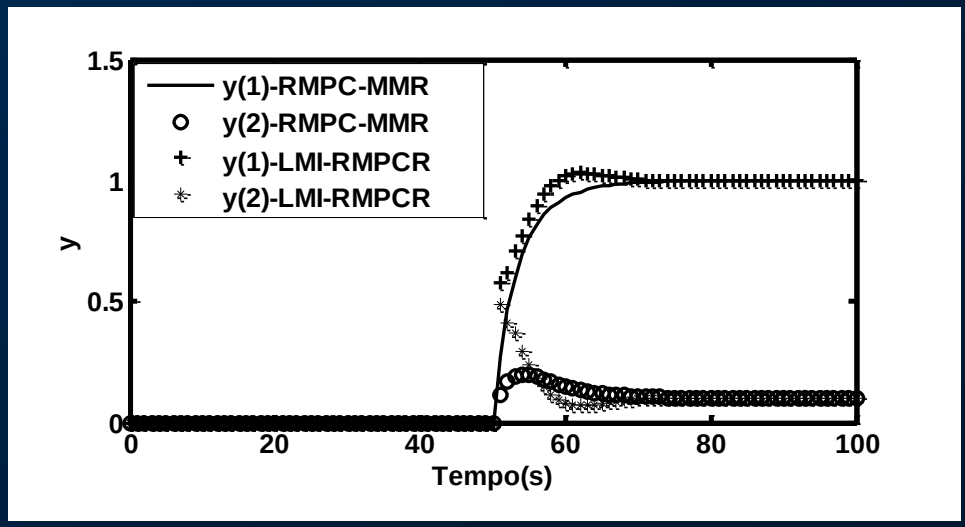
$$\mathbf{y} = \mathbf{x}$$

$$\mathbf{x}_{k+1} = \begin{bmatrix} 0{,}8187 & 0 \\ 0 & 0{,}8187 \end{bmatrix}\mathbf{x}_k + \begin{bmatrix} 0{,}1591 & -0{,}1563 \\ 0{,}1958 & -0{,}1985 \end{bmatrix}\mathbf{u}_k$$

$$\mathbf{y} = \mathbf{x}$$

Controladores:
RMPC_MMR
LMI_RMPCR





O SISTEMA EXPERIMENTAL

Sistema Experimental

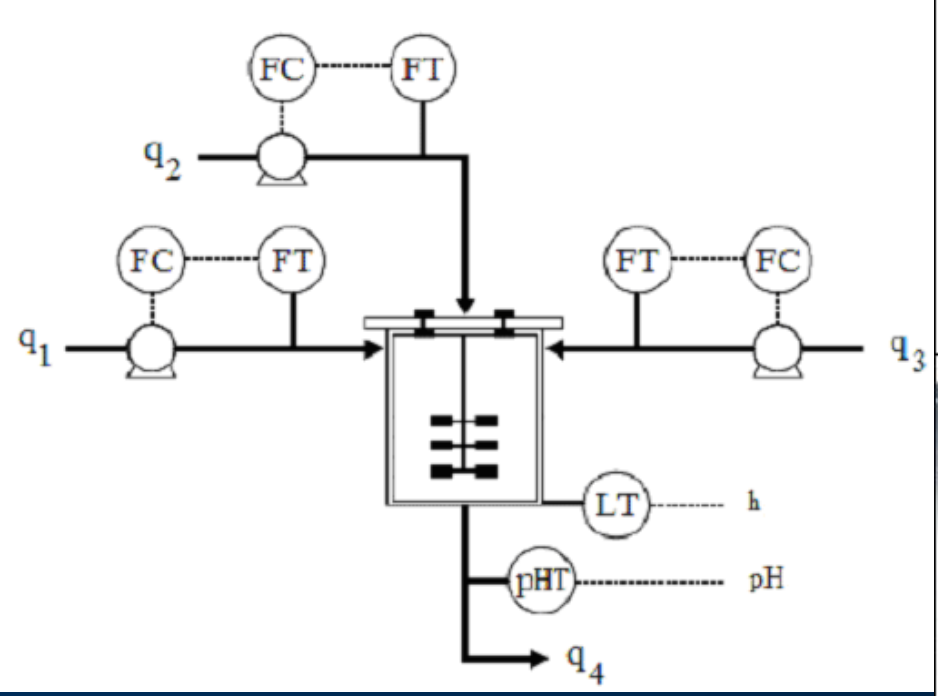
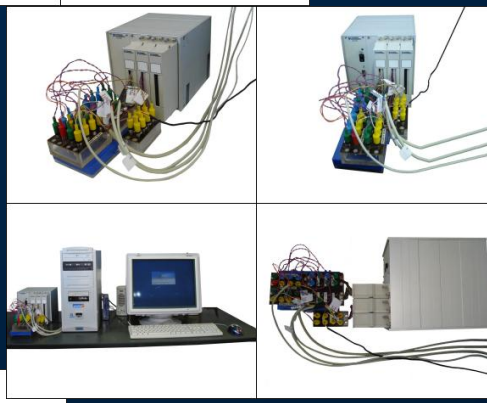


Foto: Daniel Gonçalves

Sistema Experimental



Fotos: Daniel Gonçalves

Sistema Experimental: Modelo EE

$$\frac{dh}{dt} = \frac{1}{Ar}(q1 + q2 + q3 - q4)$$

$$\frac{dW_a}{dt} = \frac{1}{Vr}(q1(W_{a1} - W_a) + q2(W_{a2} - W_a) + q3(W_{a3} - W_a))$$

$$\frac{dW_b}{dt} = \frac{1}{Vr}(q1(W_{b1} - W_b) + q2(W_{b2} - W_b) + q3(W_{b3} - W_b))$$

$$\mathbf{x} = \begin{bmatrix} h & W_a & W_b \end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix} q1 & q2 \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} h & pH \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} -c_v.p7.h^{p7-1} & 0 & 0 \\ -\dfrac{q1(W_{a1} - W_a) + q2(W_{a2} - W_a) + q3(W_{a3} - W_a)}{Arh^2} & \dfrac{-q1-q2-q3}{Arh} & 0 \\ -\dfrac{q1(W_{b1} - W_b) + q1(W_{b1} - W_b) + q1(W_{b1} - W_b)}{Arh^2} & 0 & \dfrac{-q1-q2-q3}{Arh} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \dfrac{1}{Ar} & \dfrac{1}{Ar} \\ \dfrac{(W_{a1} - W_a)}{Ar.h} & \dfrac{(W_{a2} - W_a)}{Ar.h} \\ \dfrac{(W_{b1} - W_b)}{Ar.h} & \dfrac{(W_{b1} - W_b)}{Ar.h} \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} \dfrac{1}{Ar} \\ \dfrac{(W_{a3} - W_a)}{Ar.h} \\ \dfrac{(W_{b3} - W_b)}{Ar.h} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & dydz/dWadz & dydz/dWbdz \end{bmatrix}$$

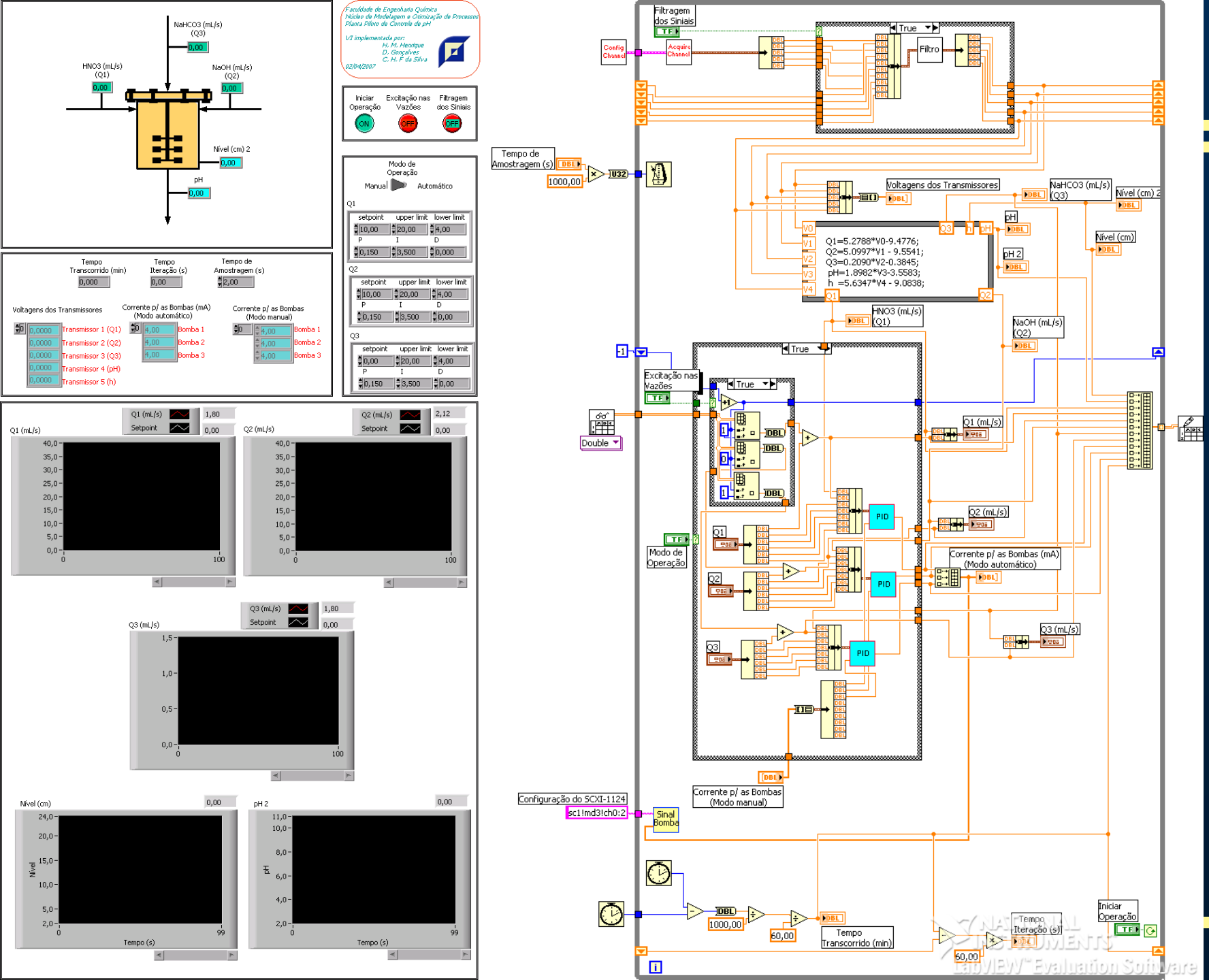
Sistema Experimental: Derivadas

$$dWadz = 1 + \frac{kw}{H^2} - \frac{W_b\left(-\frac{ka1}{H^2} - \frac{4ka1.ka2}{H^3}\right)}{1 + \frac{ka1}{H} + \frac{ka1.ka2}{H^2}} + \frac{W_b\left(\frac{ka1}{H} + \frac{2.ka1.ka2}{H^2}\right)\left(-\frac{ka1}{H^2} + \frac{2.ka1.ka2}{H^3}\right)}{\left(1 + \frac{ka1}{H} + \frac{ka1.ka2}{H^2}\right)^2}$$

$$dWbdz = \frac{\left(1 + \frac{kw}{H^2}\right)\left(1 + \frac{ka1}{H} + \frac{ka1.ka2}{H^2}\right)}{\frac{ka1}{H} + \frac{2.ka1.ka2}{H^2}} - \frac{\left(H - \frac{kw}{H} - W_a\right)\left(1 + \frac{ka1}{H} + \frac{ka1.ka2}{H^2}\right)\left(\frac{-ka1}{H^2} - \frac{4.ka1.ka2}{H^3}\right)}{\left(\frac{ka1}{H} + \frac{2.ka1.ka2}{H^2}\right)^2}$$

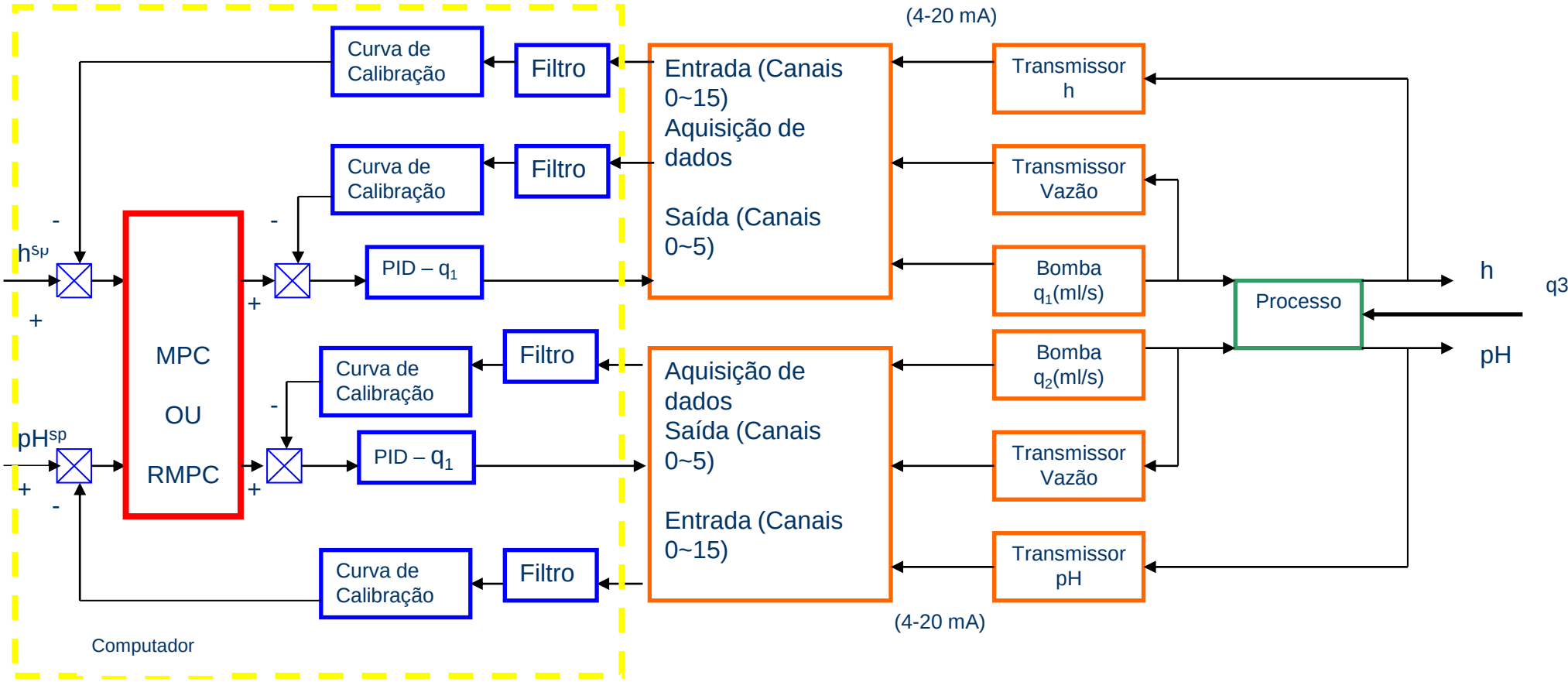
$$+ \frac{\left(H - \frac{kw}{H} - W_a\right)\left(\frac{-ka1}{H^2} - \frac{2.ka1.ka2}{H^3}\right)}{\frac{ka1}{H} + \frac{2.ka1.ka2}{H^2}}$$

$$dydz = -\log_{10}\frac{e}{H^+}$$

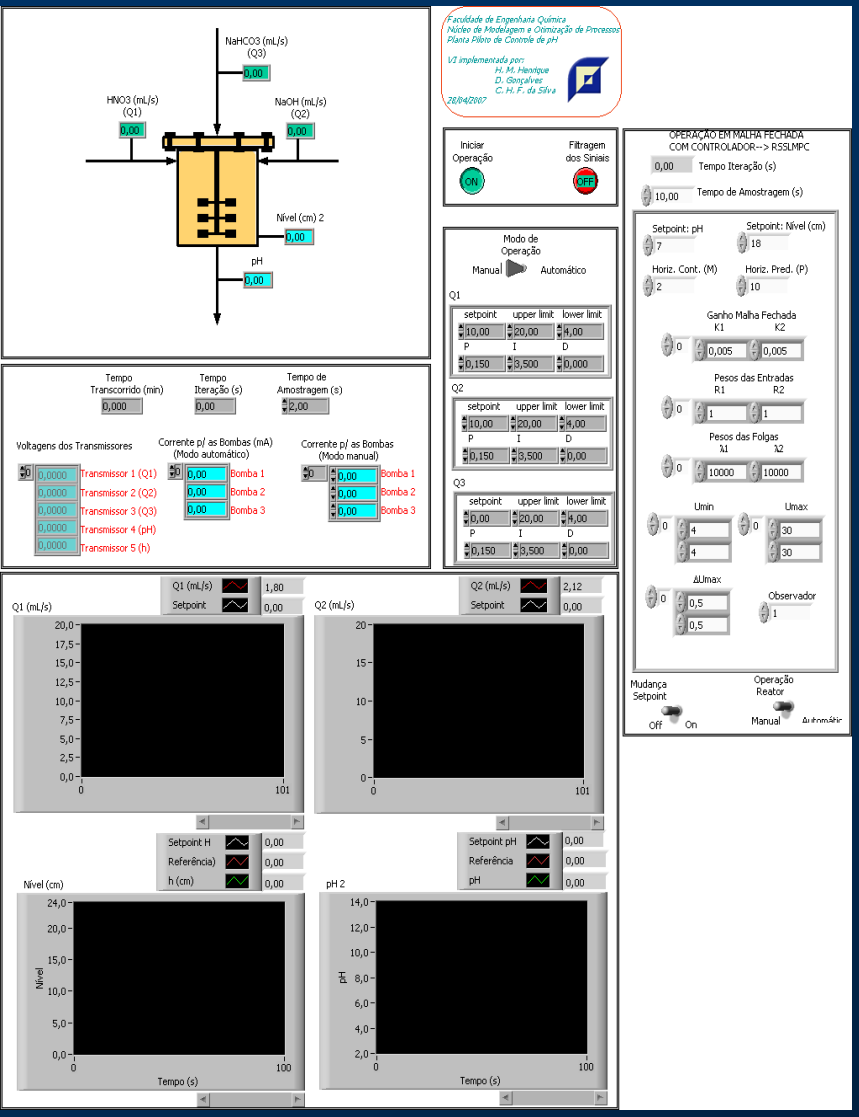


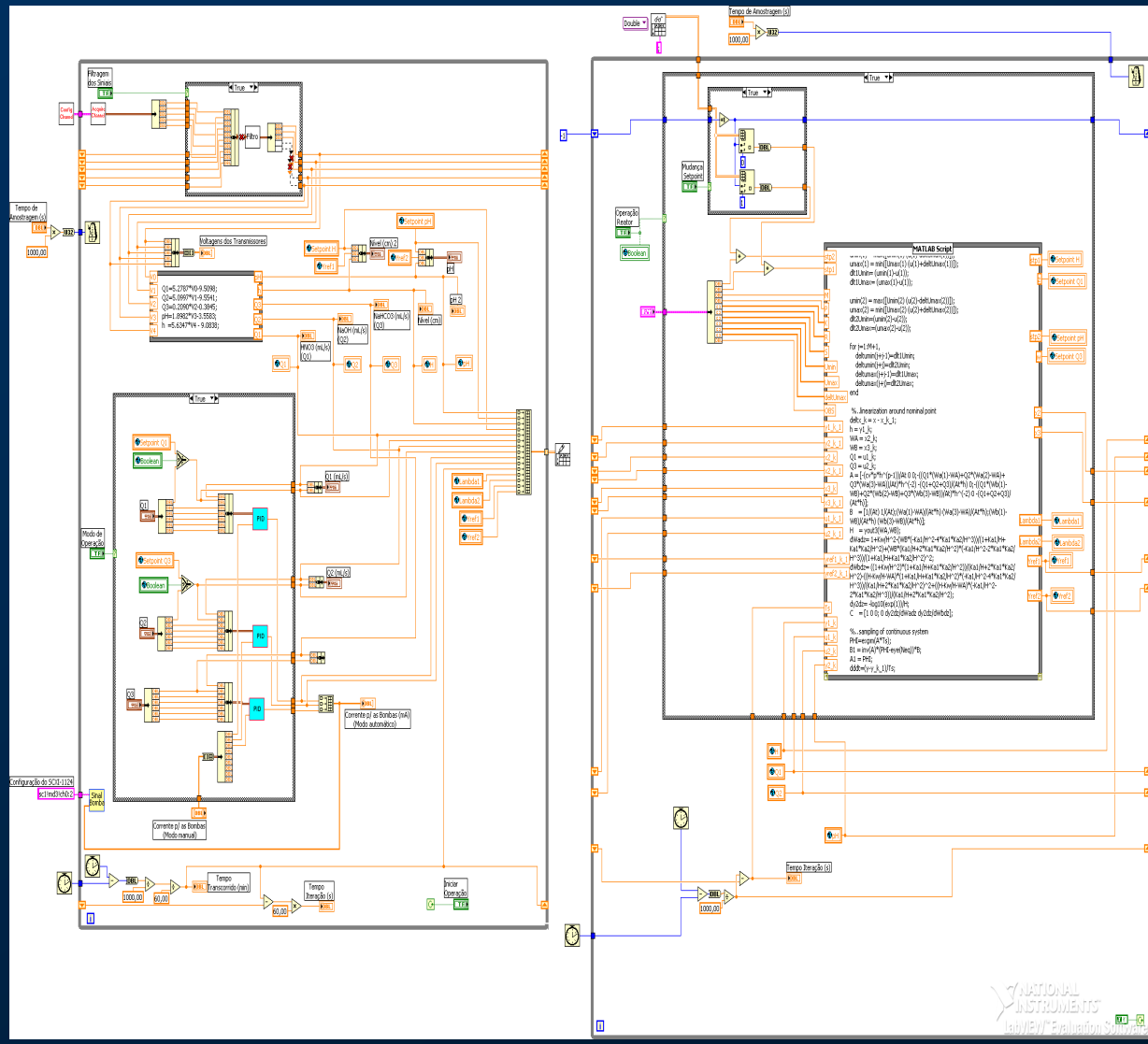
V I M A L H A A B E R T A

Sistema Experimental: Diagrama de Operação

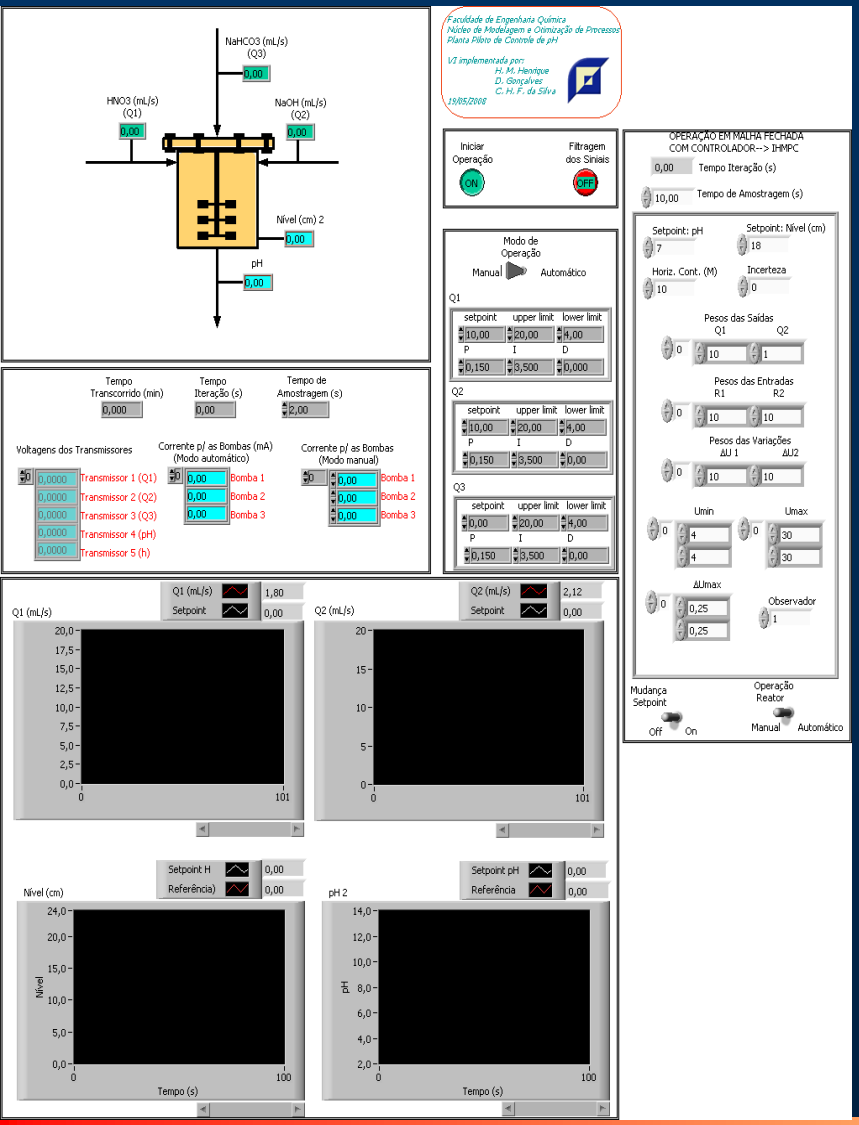


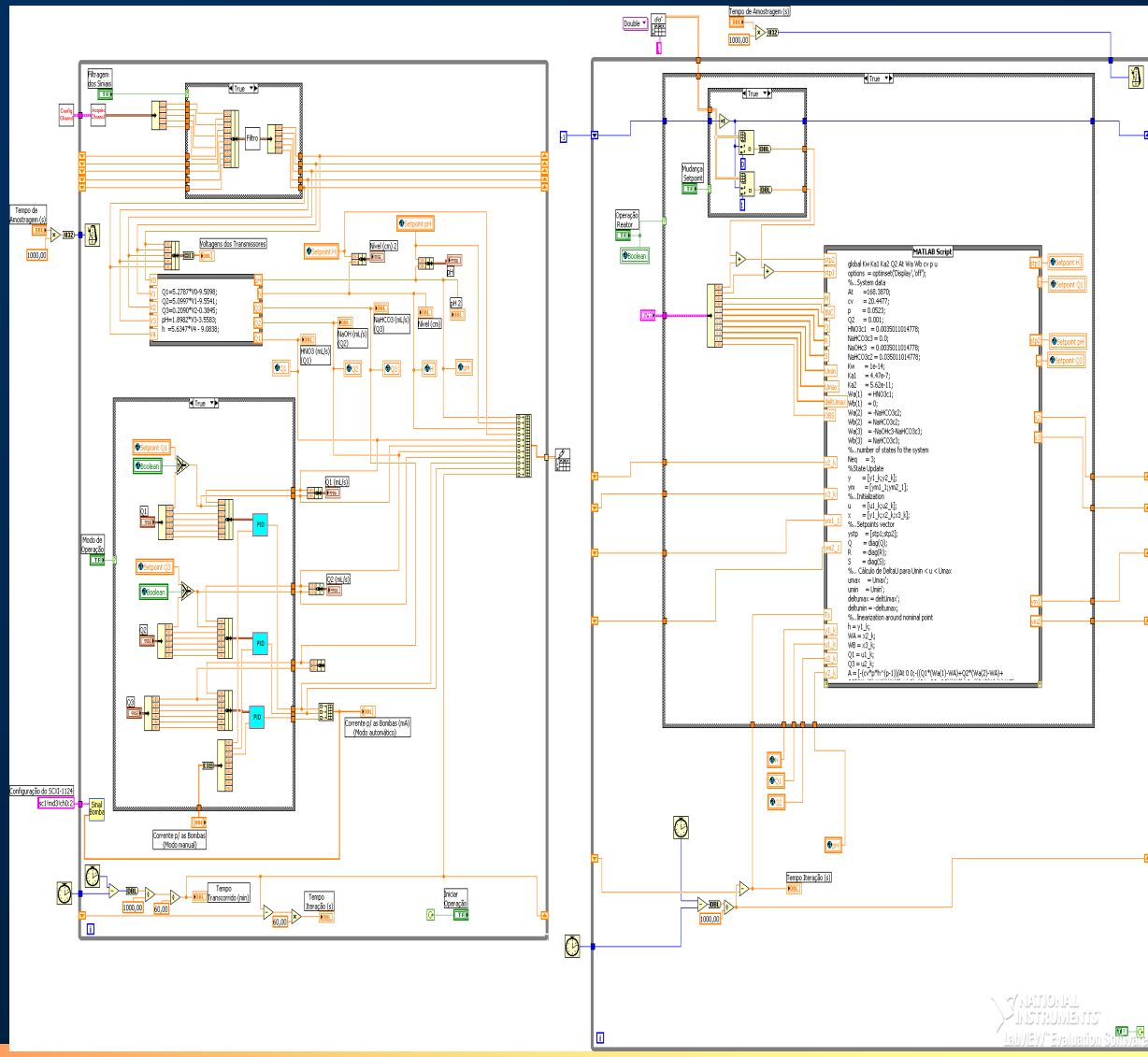
Sistema Experimental: VI - Malha Fechada - RSSLMPC



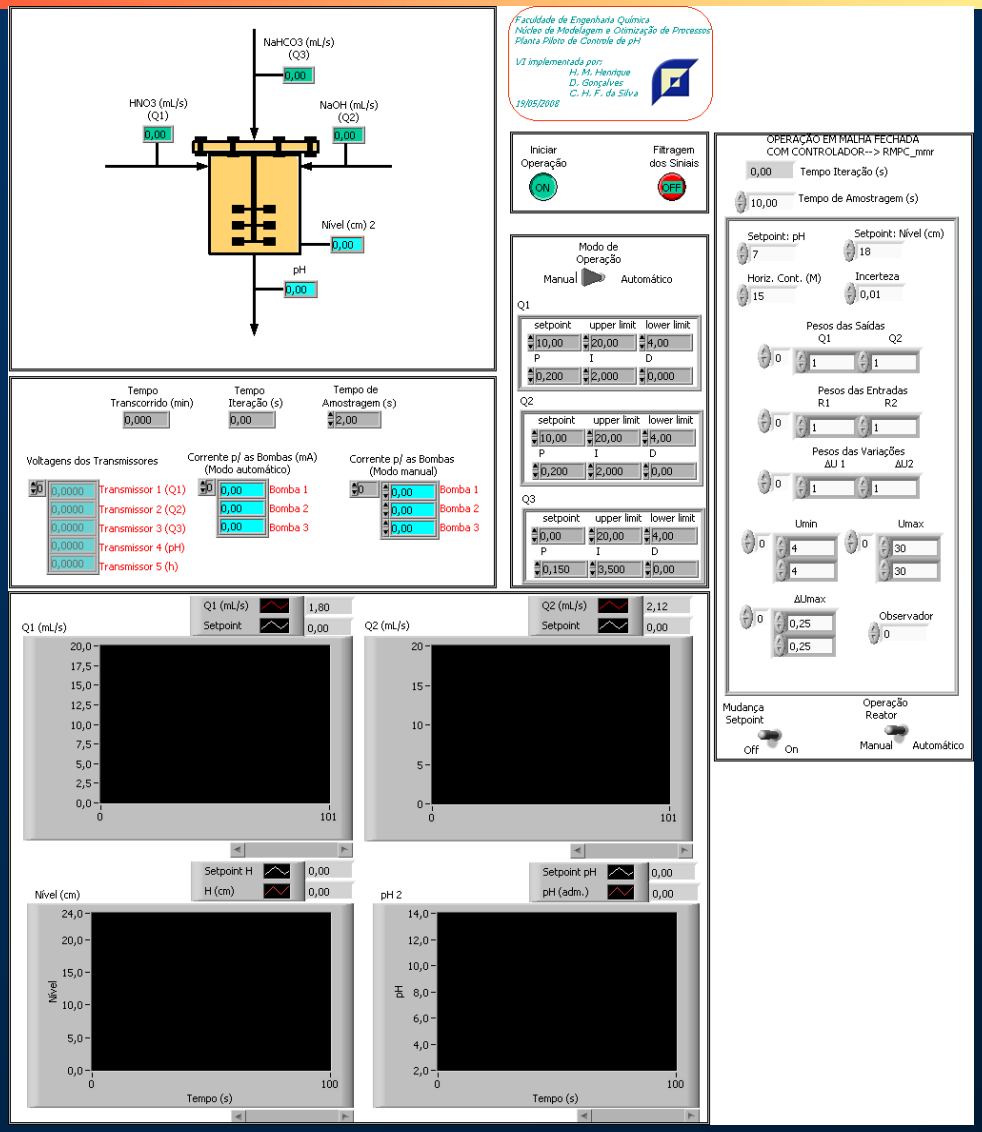


Sistema Experimental: VI - Malha Fechada - IHMPC





Sistema Experimental: VI – Malha Fechada – RMPC's



Sistema Experimental: VI – Malha Fechada

- Estados;
- Observador em malha aberta adicionado à FO;

$$\mathbf{d} = \mathbf{y}_{\text{Plant}} - \mathbf{y}_{\text{Mod.}}$$

- Modificação para trabalho no PO:

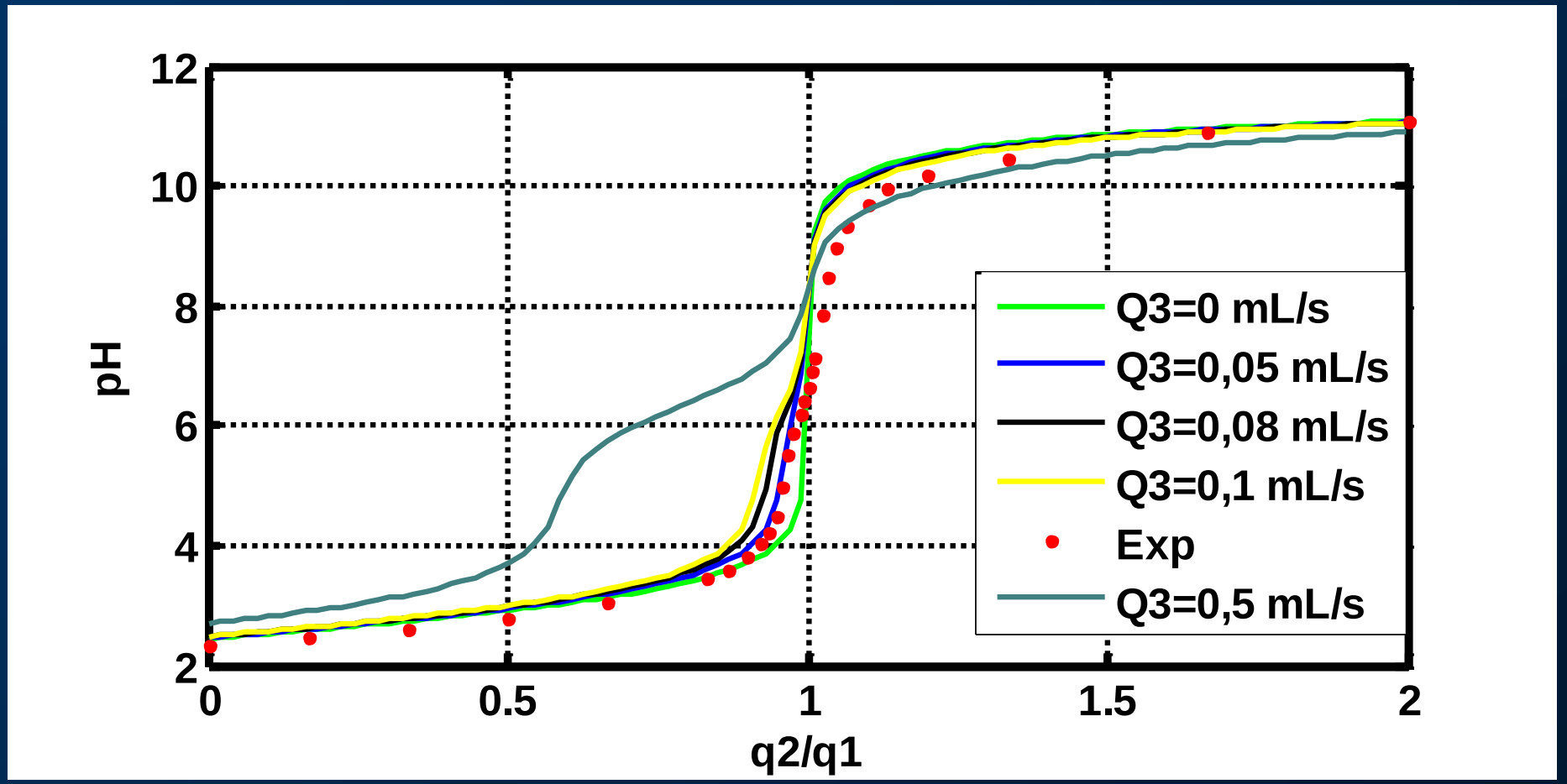
$$\frac{d\hat{\mathbf{x}}'}{\mathrm{dt}} = \begin{bmatrix} \mathbf{A}_{k-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{f}_{k-1} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}}' \\ 1 \end{bmatrix} + \mathbf{B}_{k-1}\mathbf{u}'$$

- FO na forma LTV:

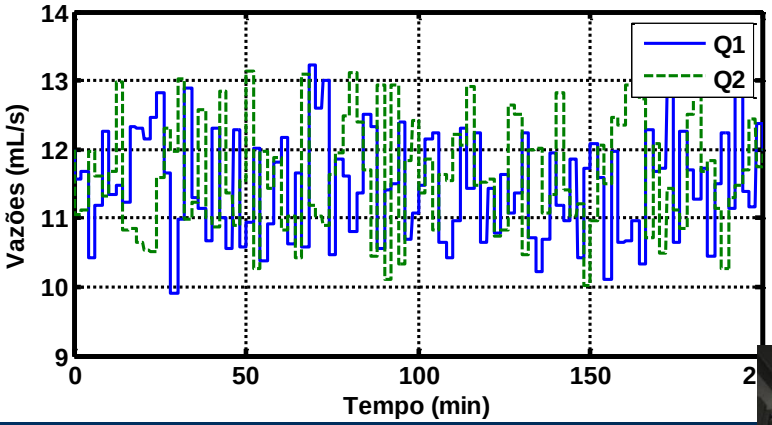
$$J_k = \min_{\Xi} \Xi^T \mathbf{H}_k \Xi + 2\Xi^T (\mathbf{G}_k \mathbf{z}_k - \mathbf{F}_k \mathbf{u}_{k-1})$$

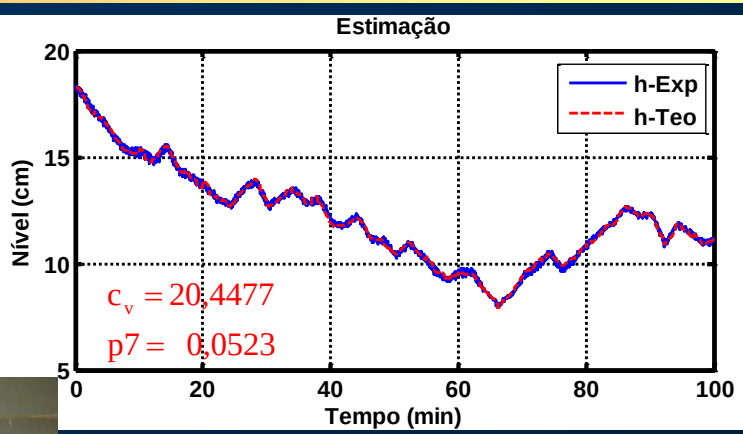
RESULTADOS EXPERIMENTAIS

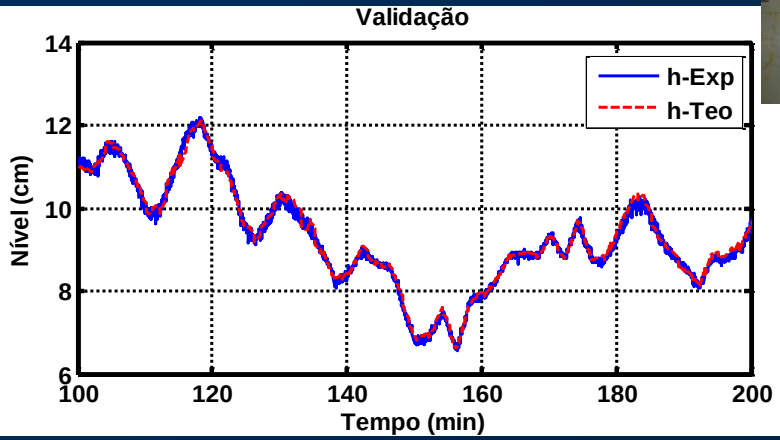
Sistema Experimental: Ganho Estático

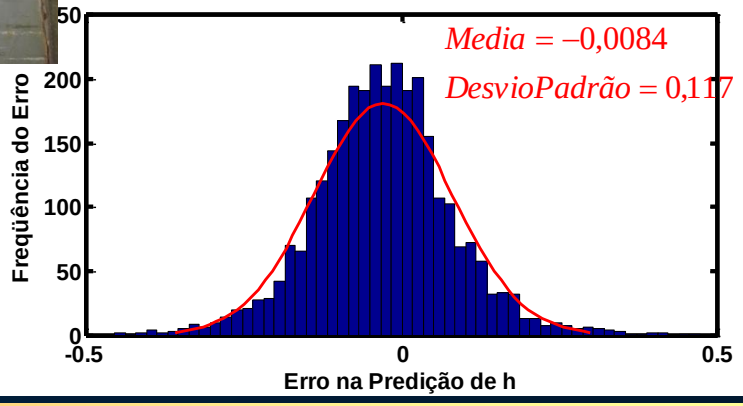


Sistema Experimental: Estimação de parâmetros da válvula



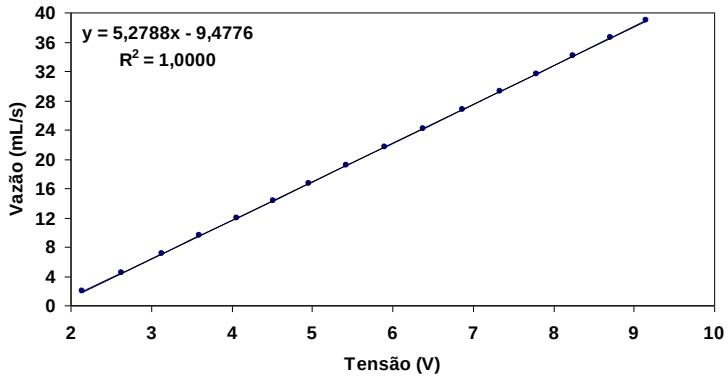


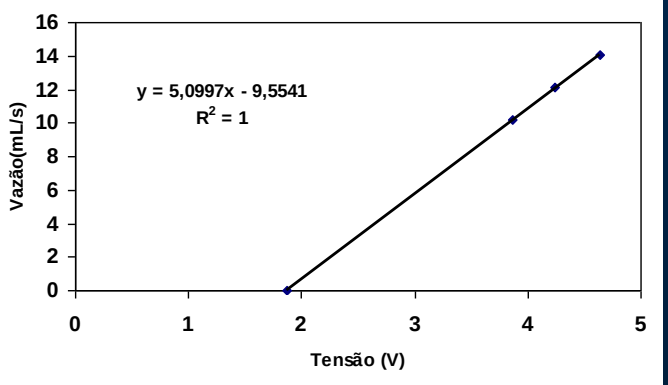


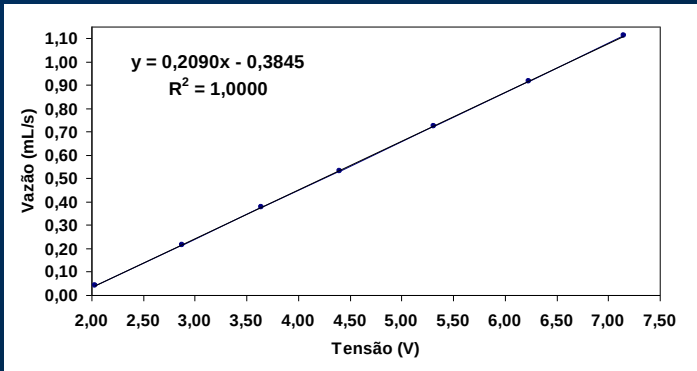


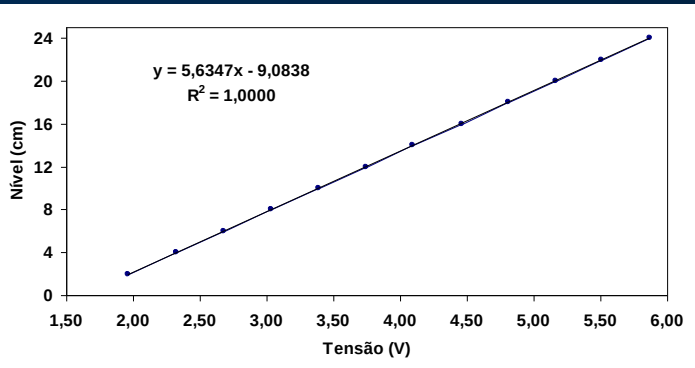
Sistema Experimental: Calibração

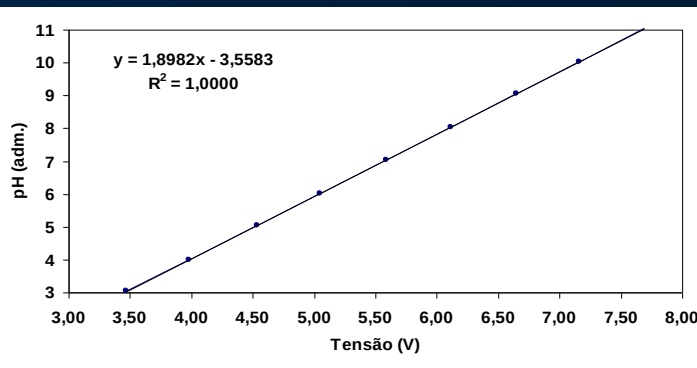
$$q1 = 5.2788 * V0 - 9.4776$$
$$q2 = 5.0997 * V1 - 9.5541$$
$$q3 = 0.2090 * V2 - 0.3845$$
$$pH = 1.8982 * V3 - 3.5583$$
$$h = 5.6347 * V4 - 9.0838$$



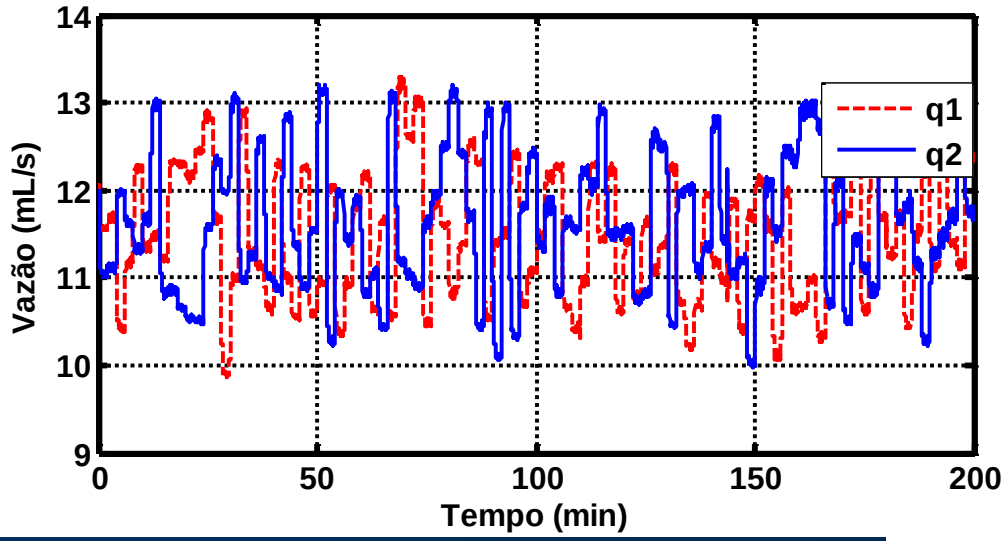


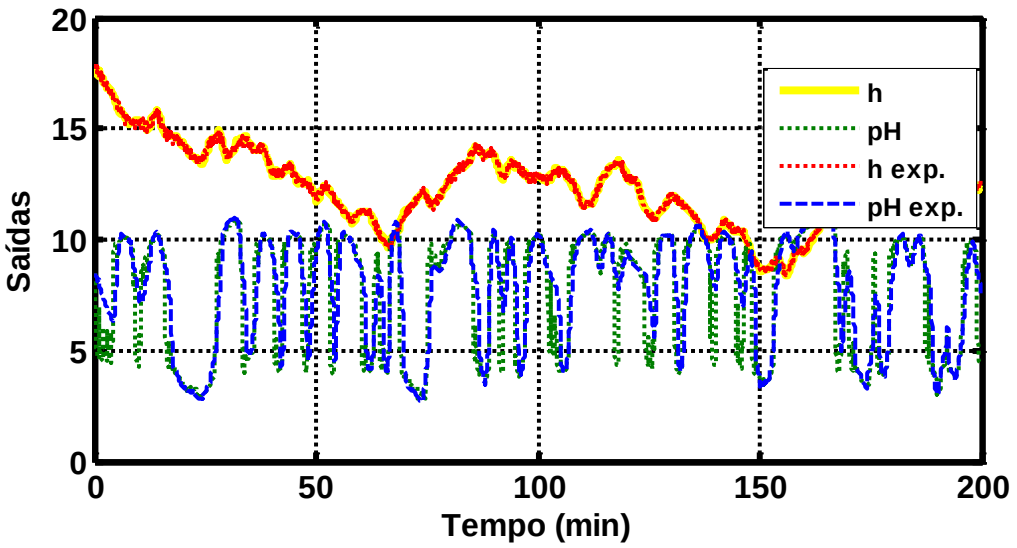




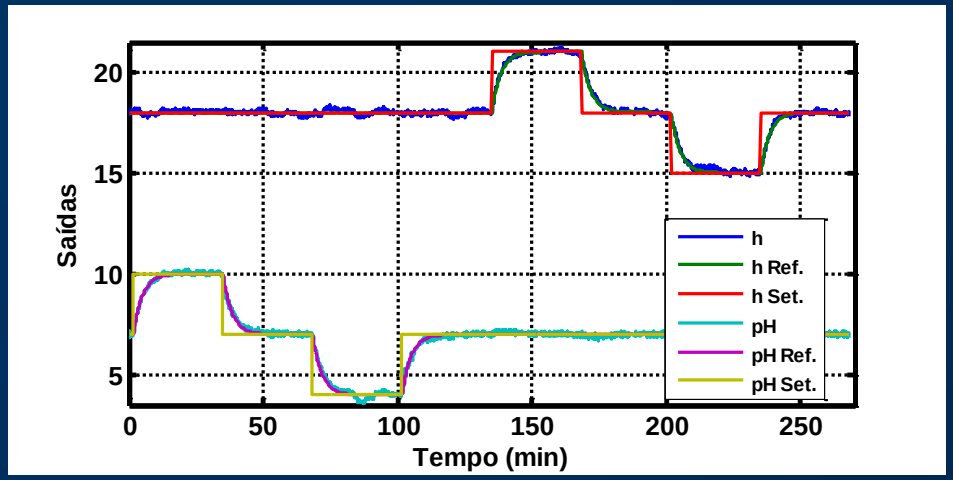


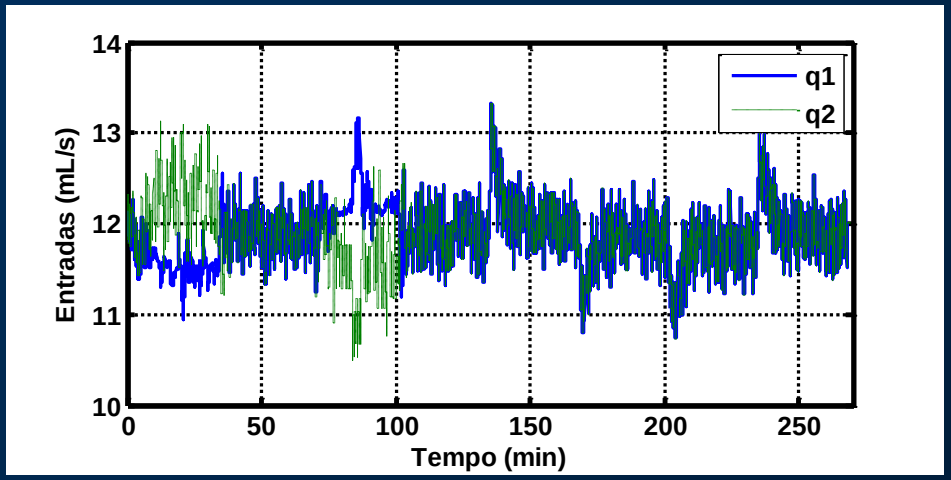
Sistema Experimental: Modelo x Experimental

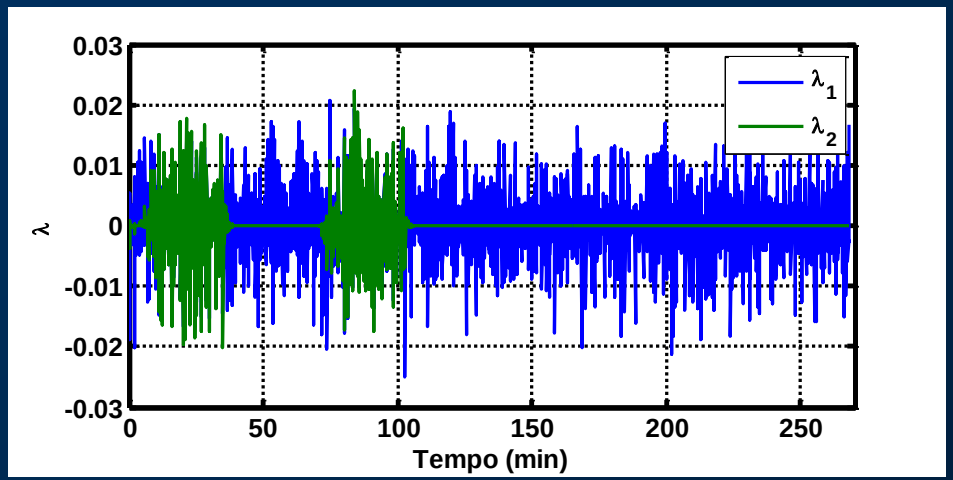


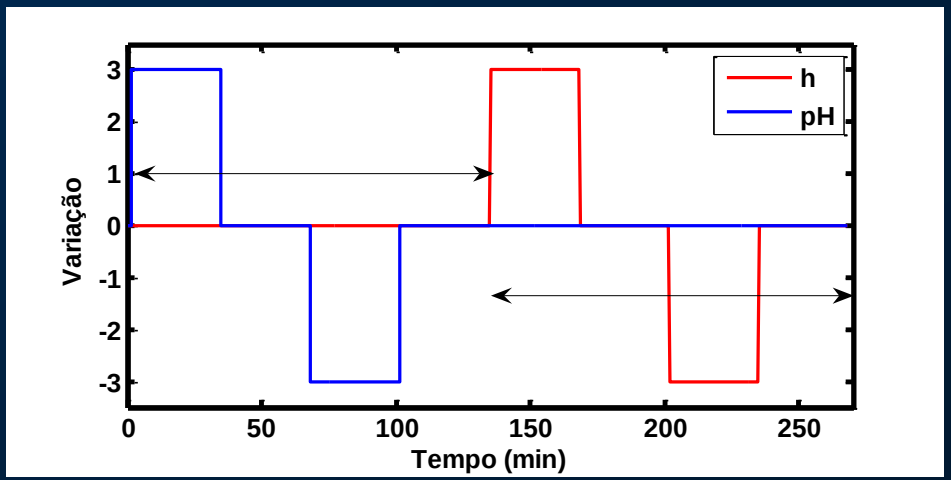


Sistema Experimental: Simulação RSSLMPC

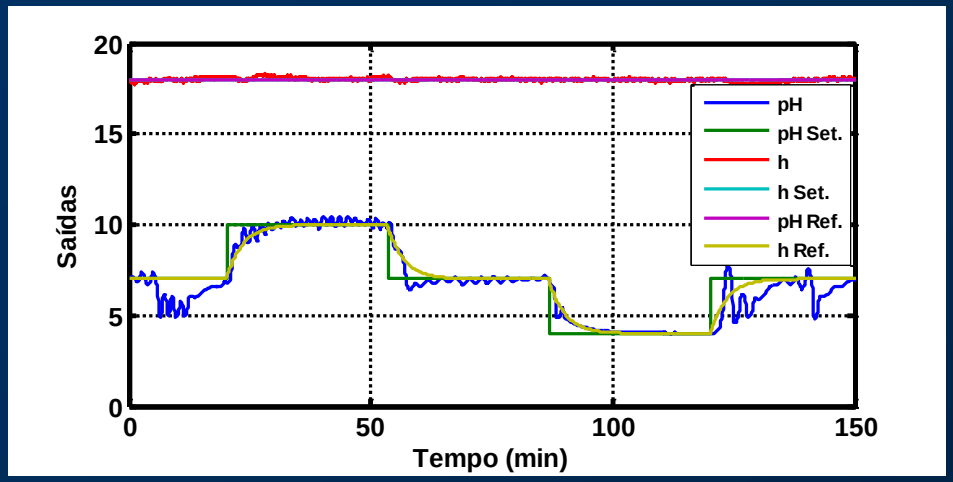


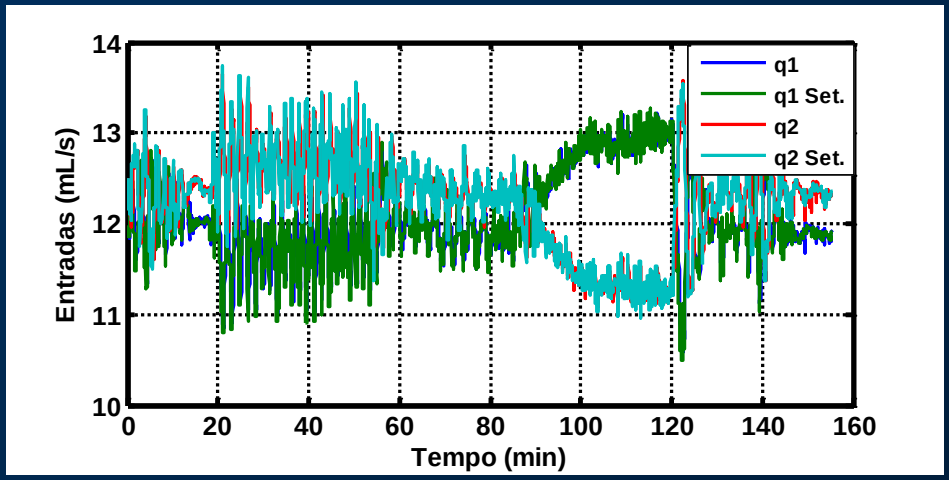


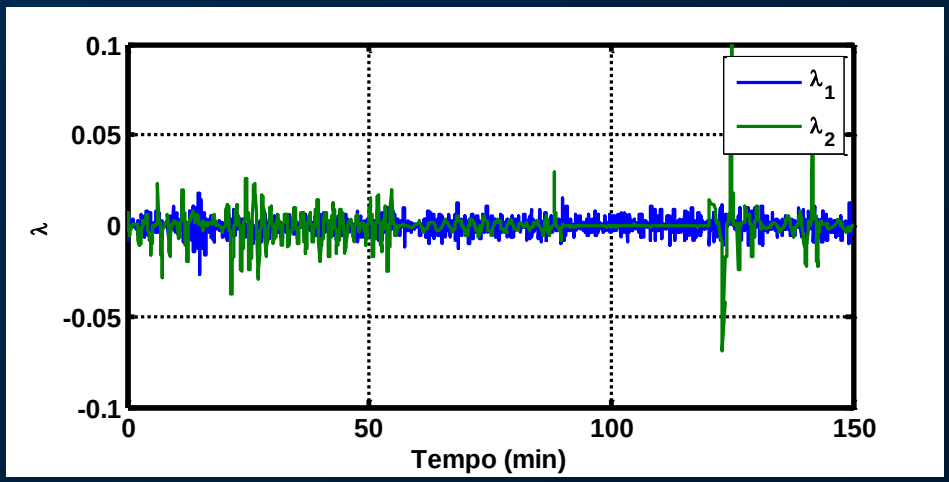




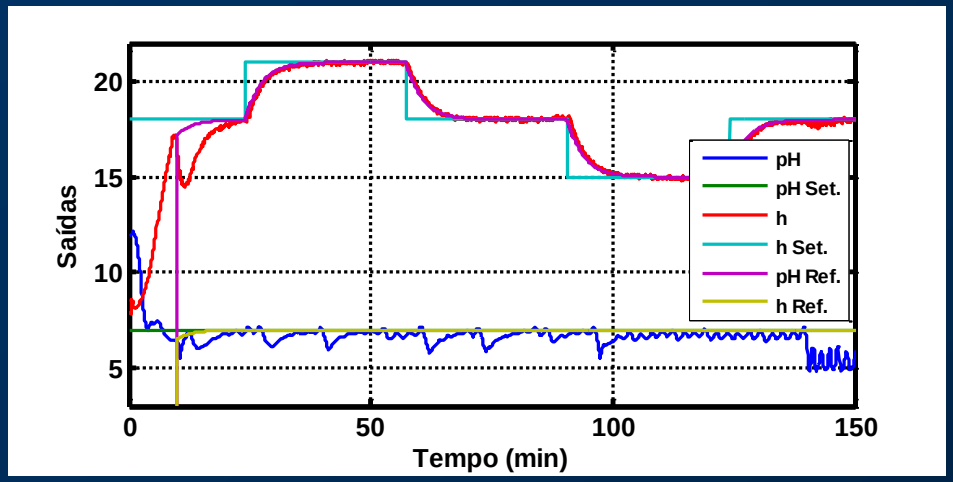
Sistema Experimental: Operação em Tempo Real - RSSLMPC

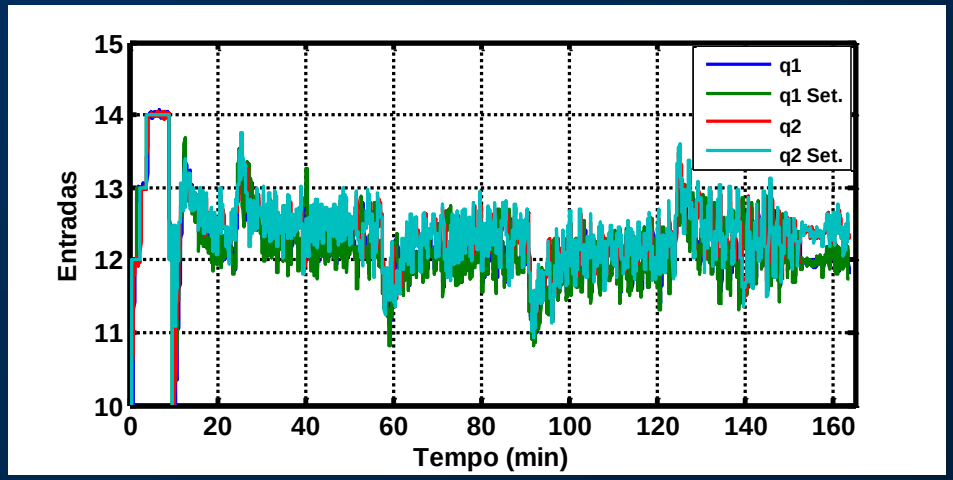


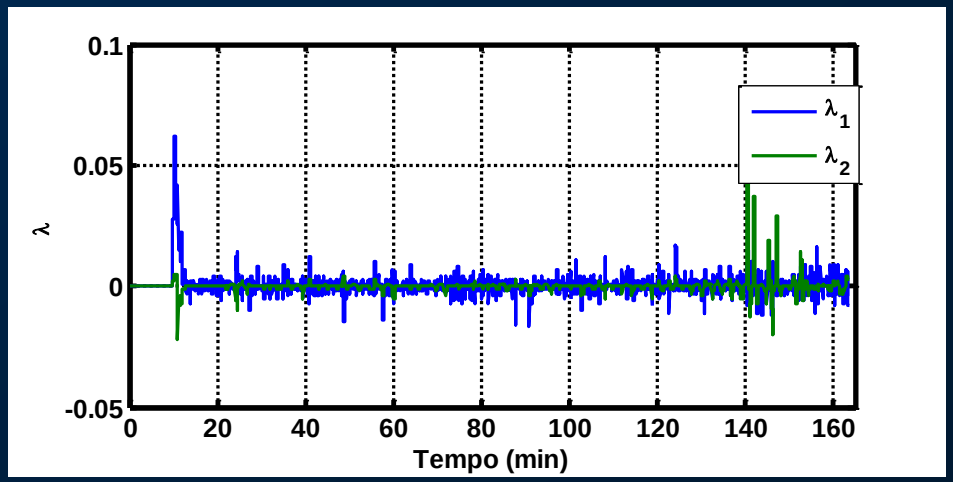




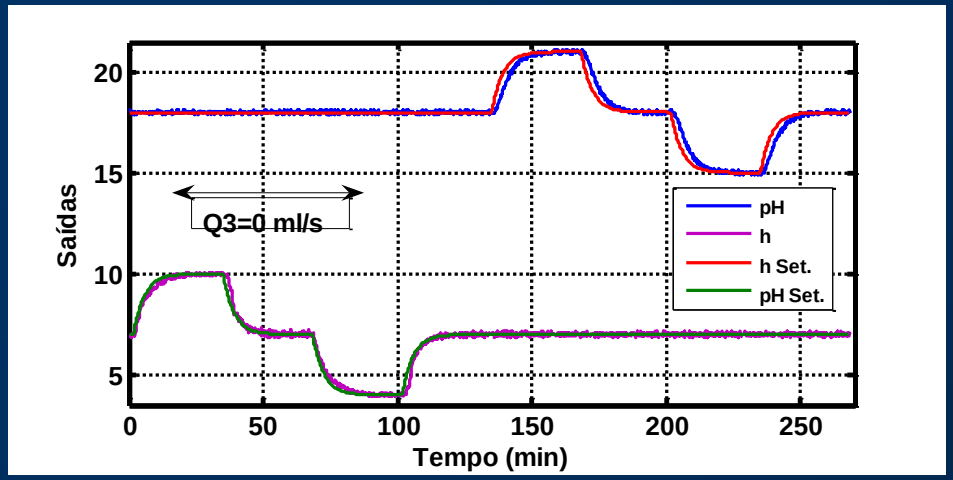
Sistema Experimental: Operação em Tempo Real - RSSLMPC

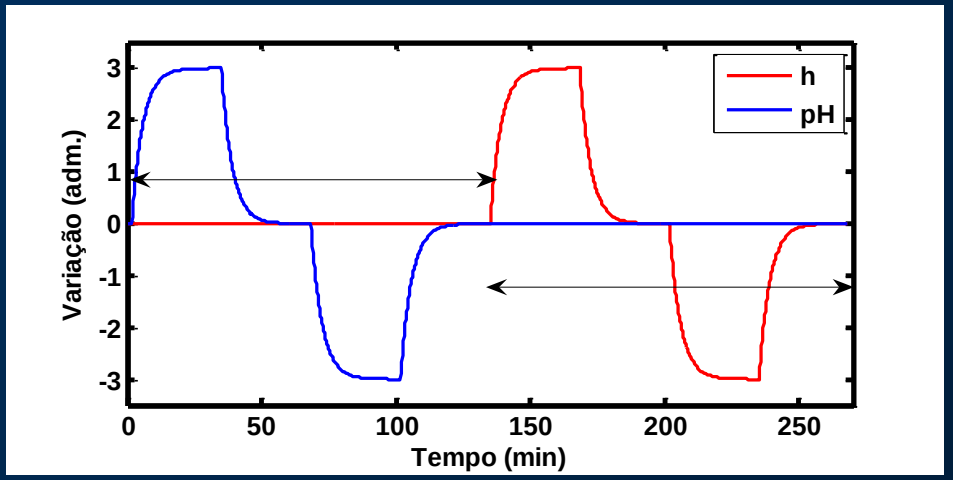


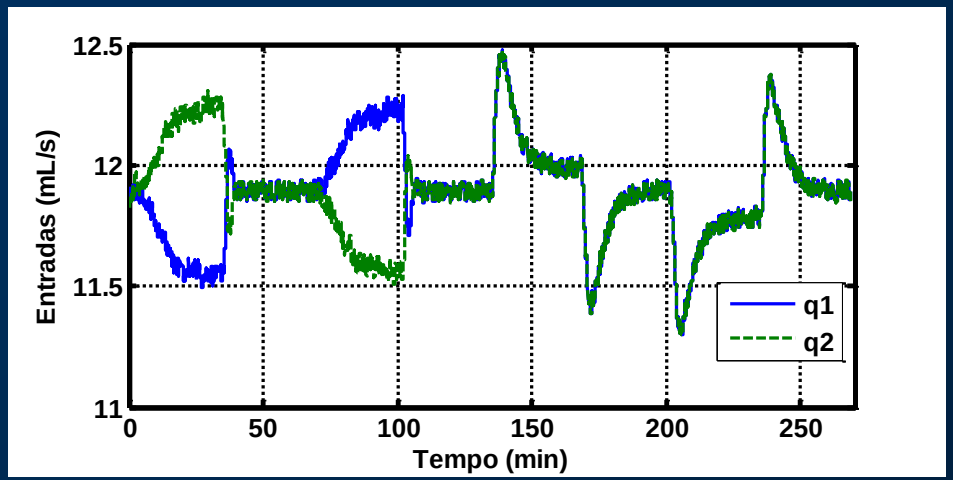


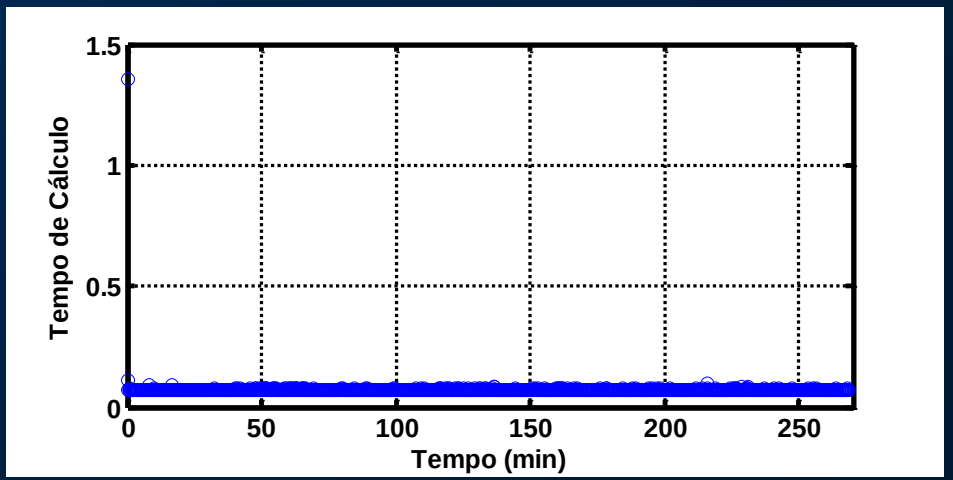


Sistema Experimental: Simulação IHMPC

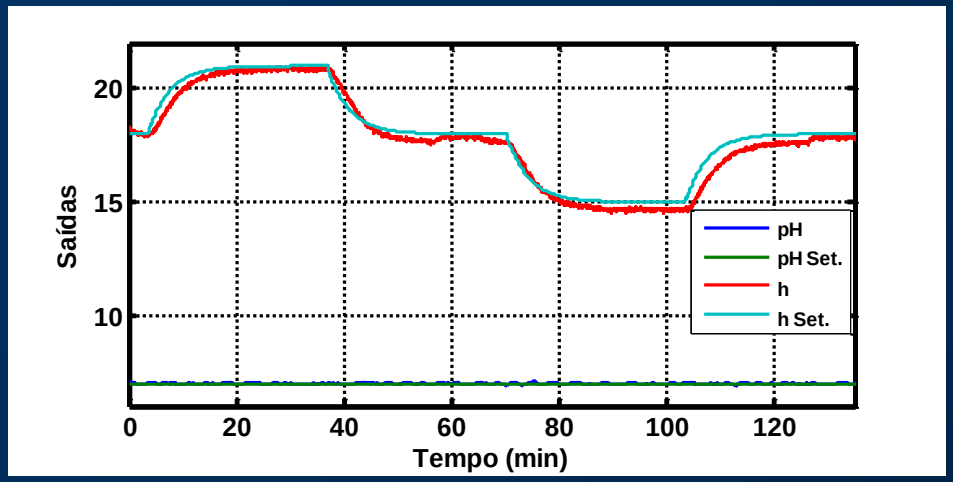


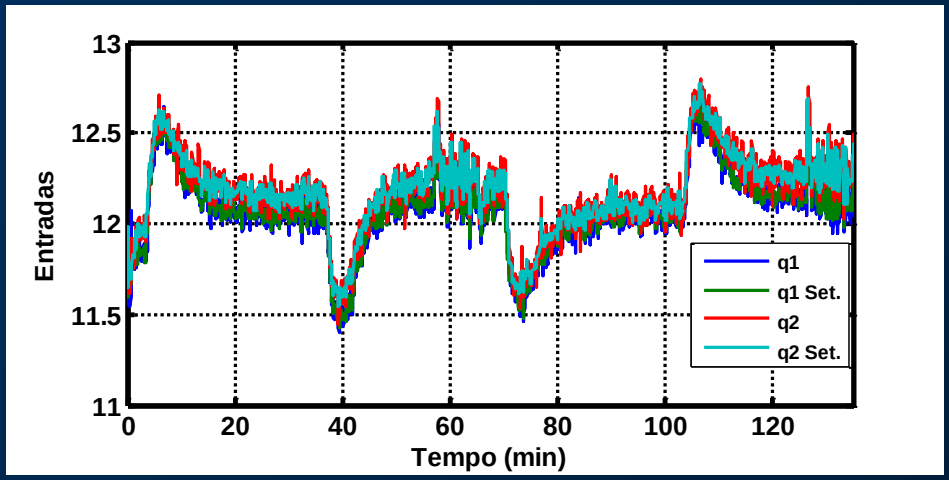


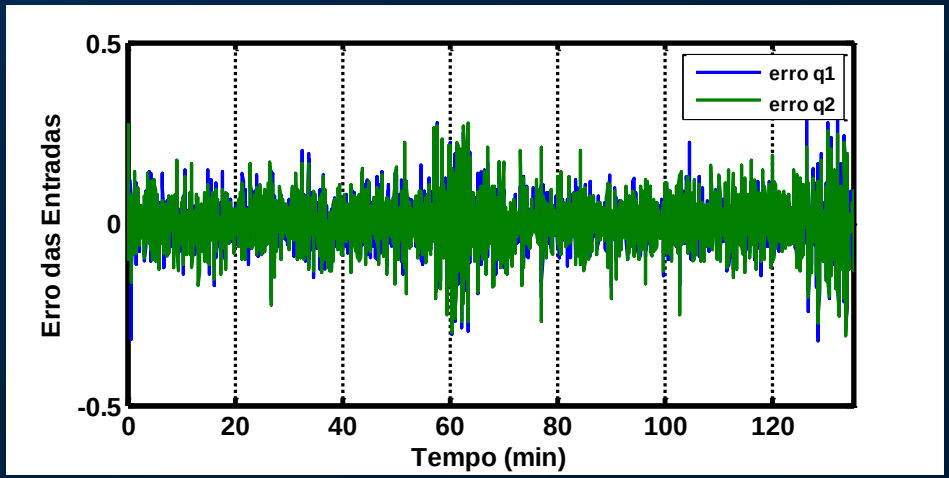




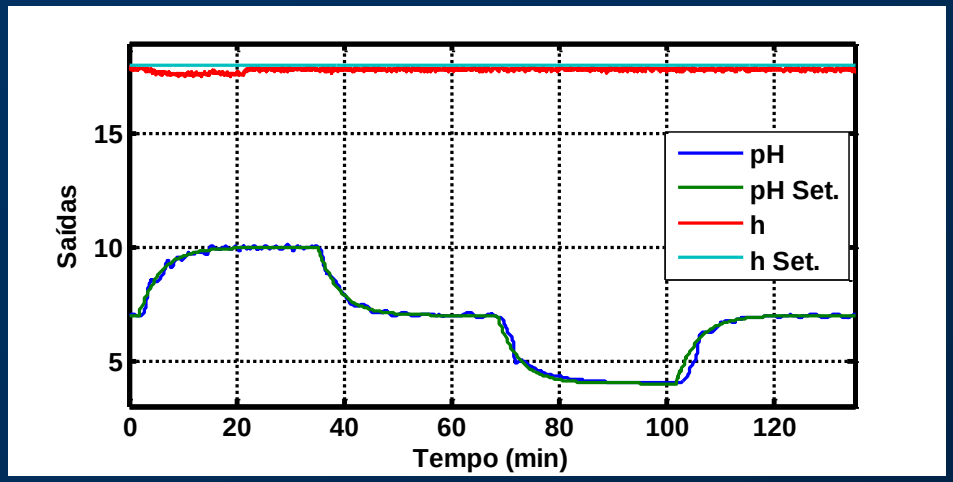
Sistema Experimental: Operação em Tempo Real – IHMPC

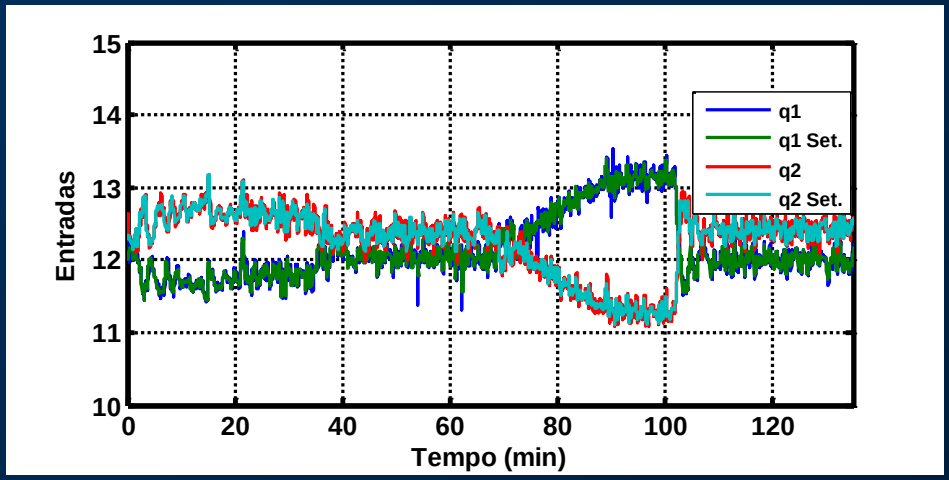


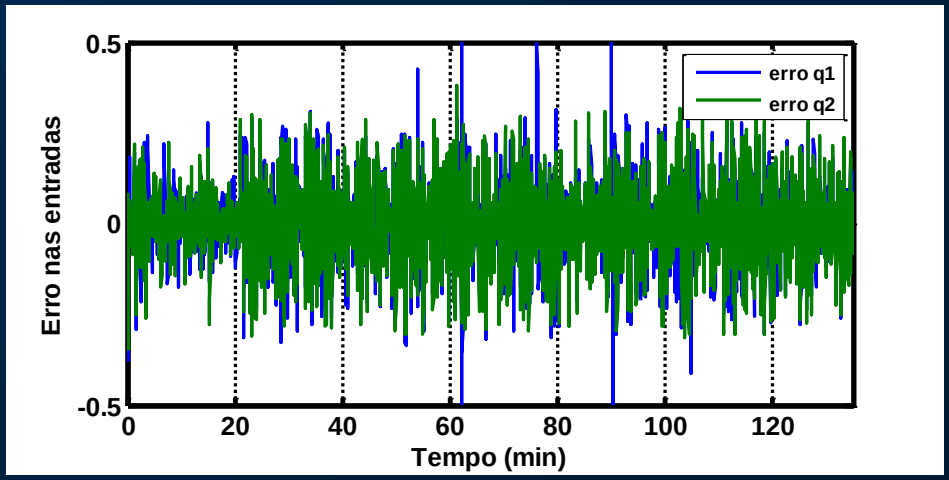




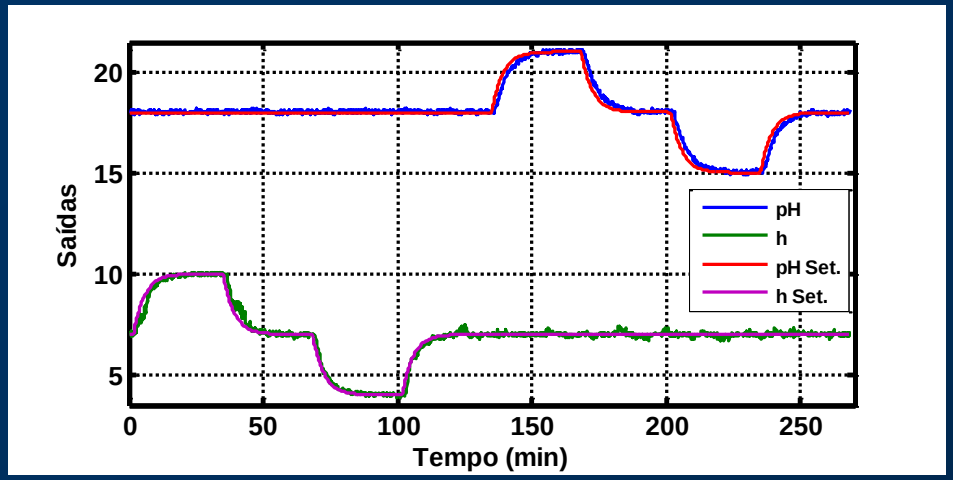
Sistema Experimental: Operação em Tempo Real - IHMPC

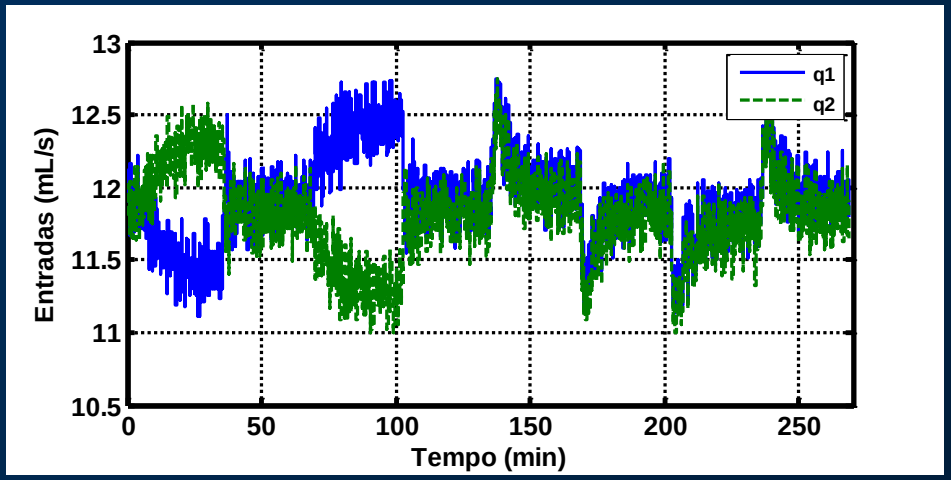


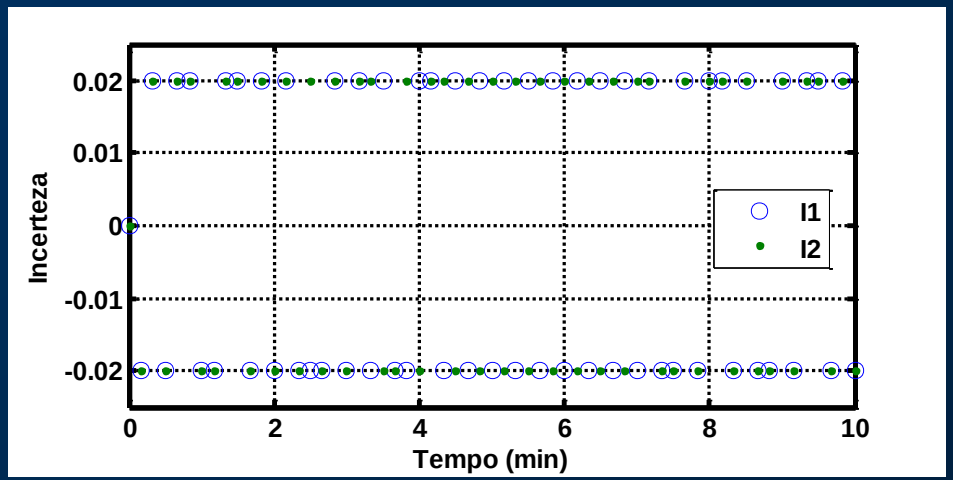


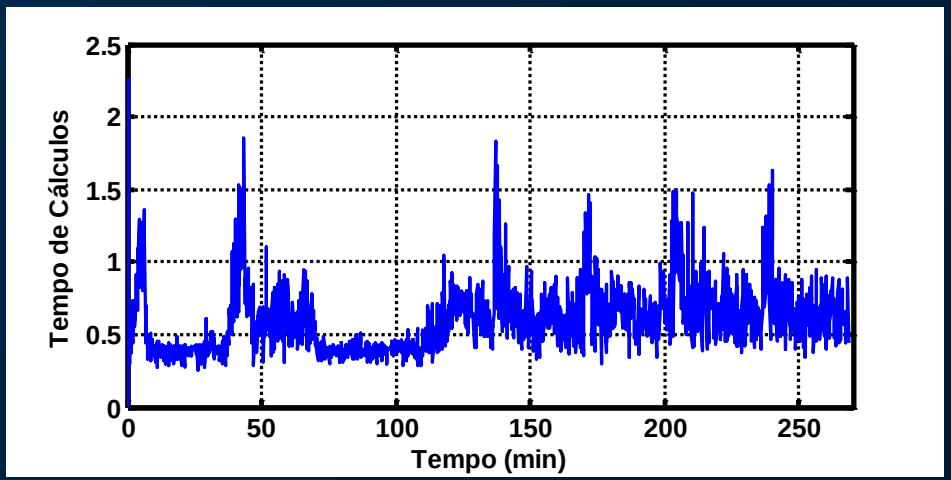


Sistema Experimental: Simulação – RMPC_MMR

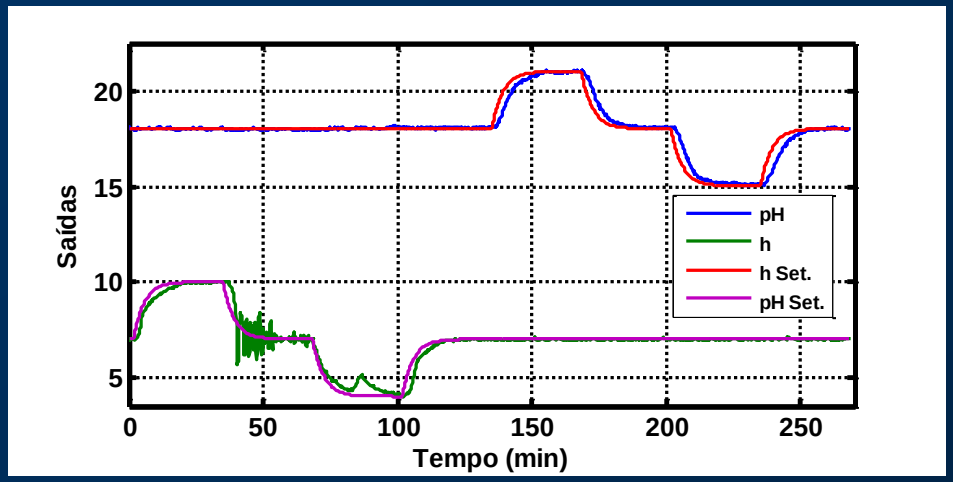


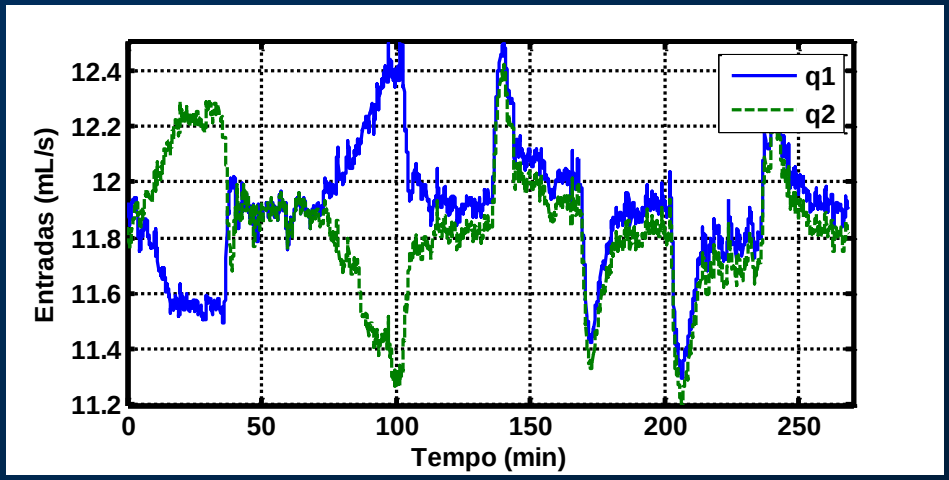


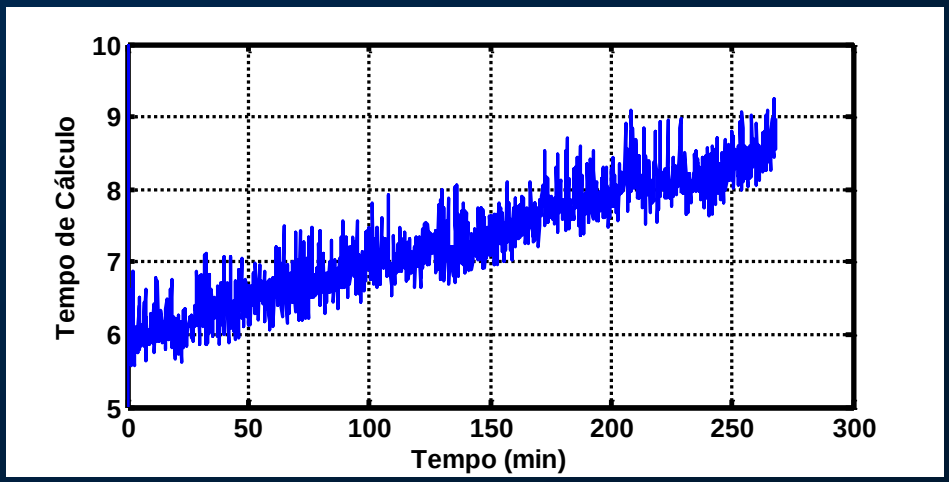




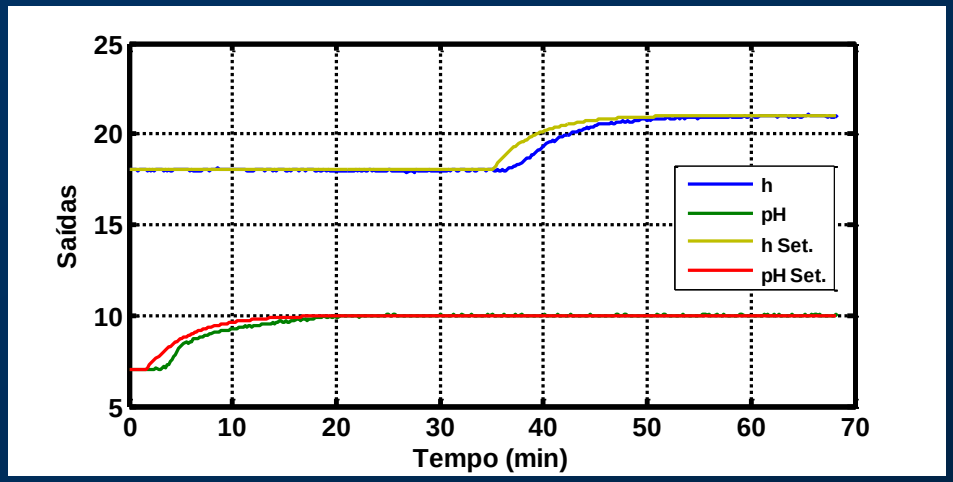
Sistema Experimental: Simulação – LMI_RMPCR - nominal

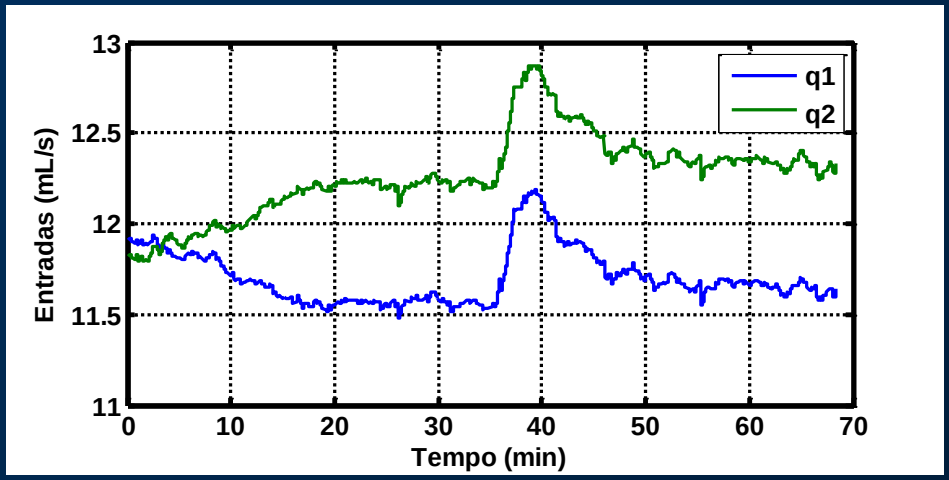


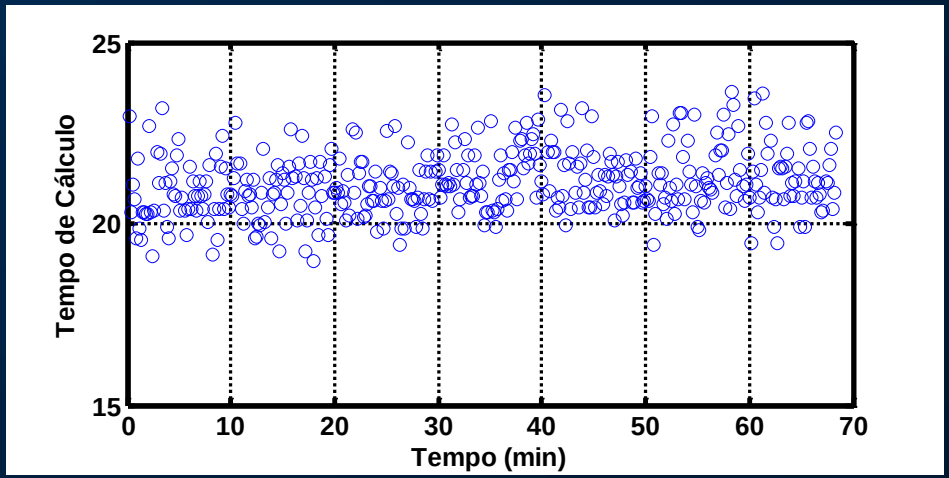




Sistema Experimental: Simulação – LMI_RMPCR - politopo



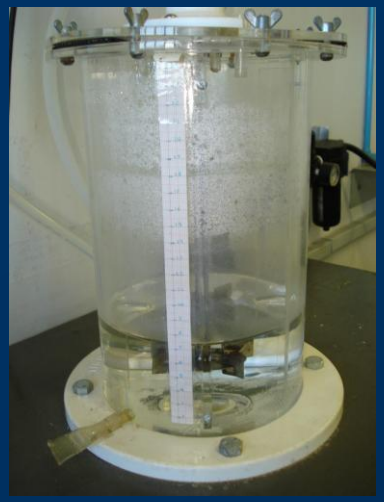


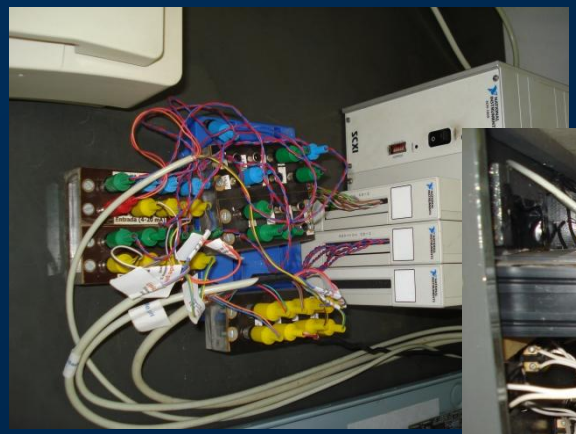


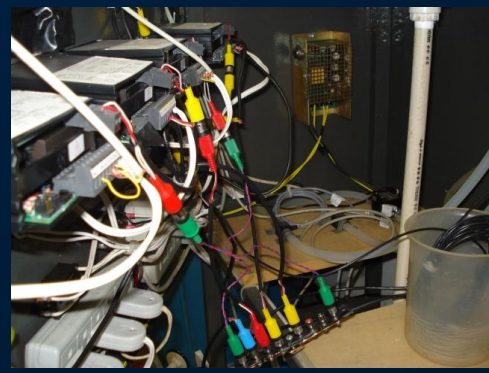
Quadro Comparativo

	PI	IHMPC	RMPC_MMTV	LMI_RMPC	RSSLMPC	LMI_RMPC nom.	RMPC_MMR	LMI_RMPCR
CSTR	Lento	☑	☑	-	-	-	☑	☑
MI/PE	-	-	-	☑	-	-	-	☑
MC	-	-	-	-	-	-	☑	☑
Exp. Sim.		☑	-	-	☑	☑	☑	
Exp. Real	-	☑	-	-	☑	-	-	-

Sistema Experimental: Problemas







Sistema Experimental: Problemas

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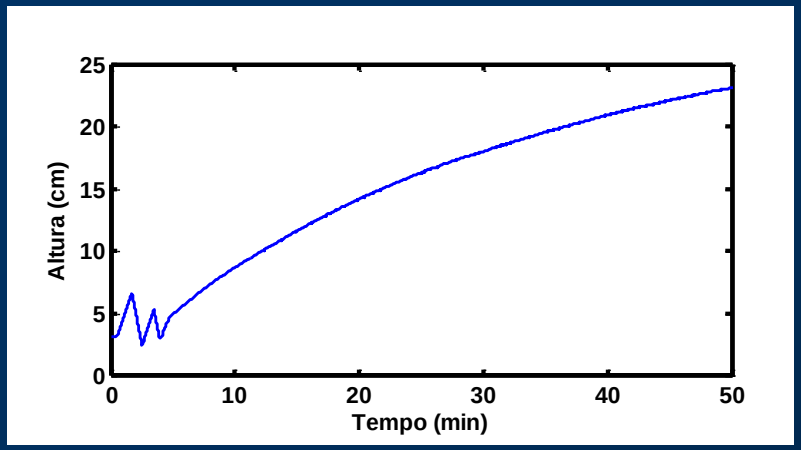
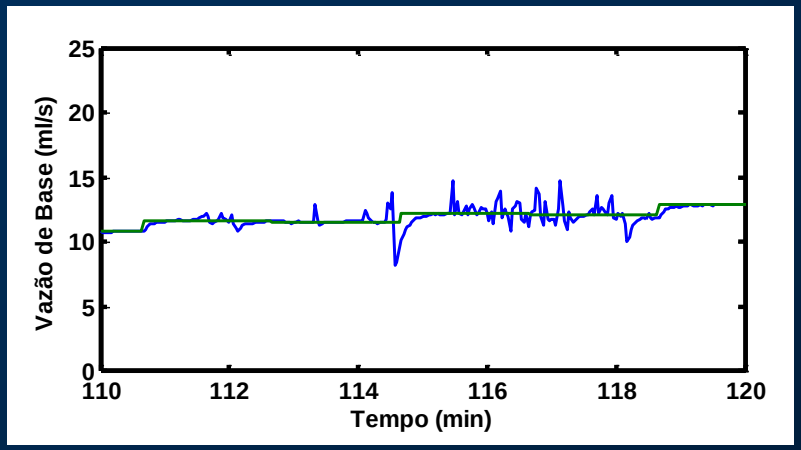
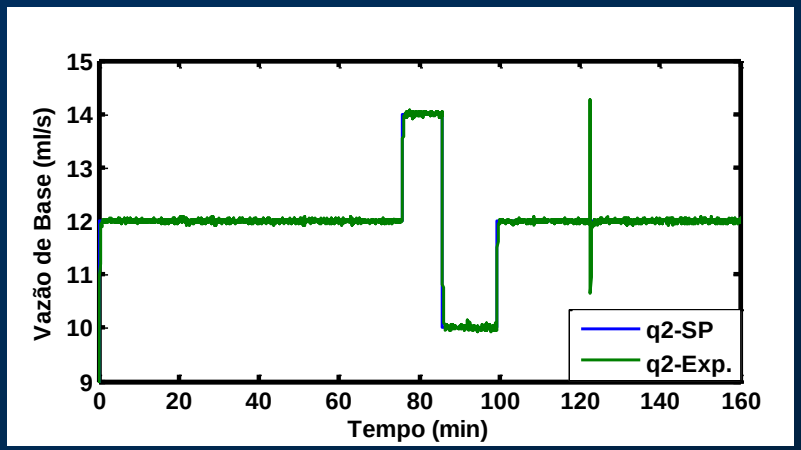
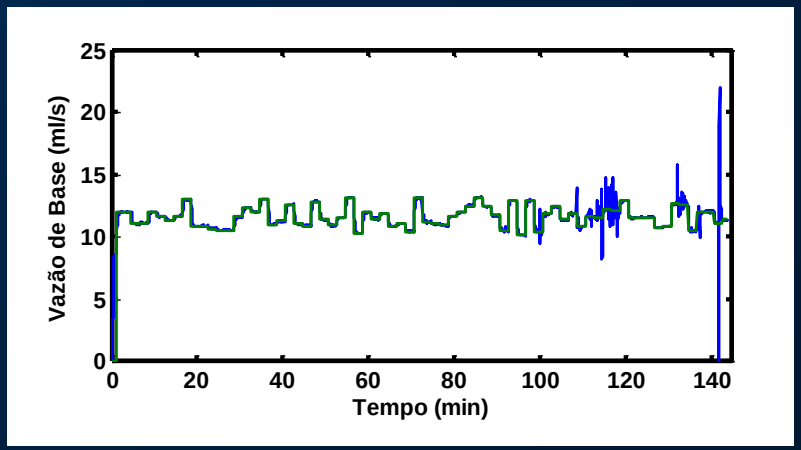
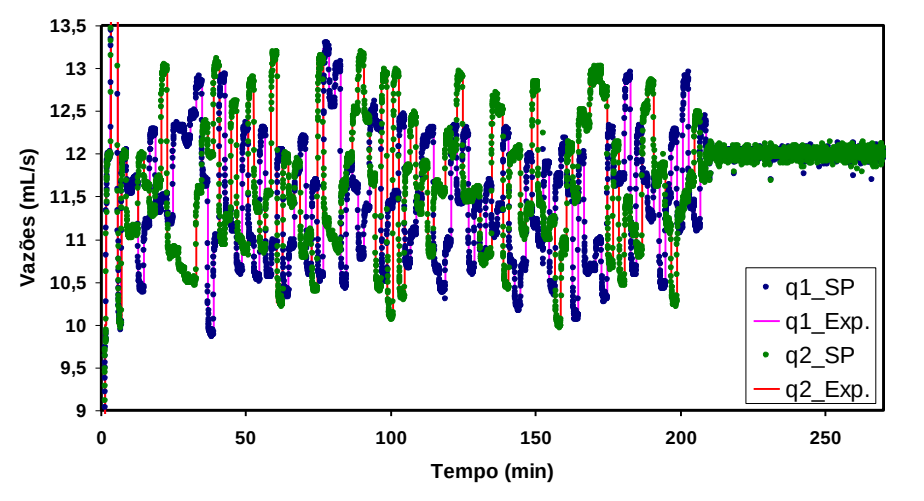


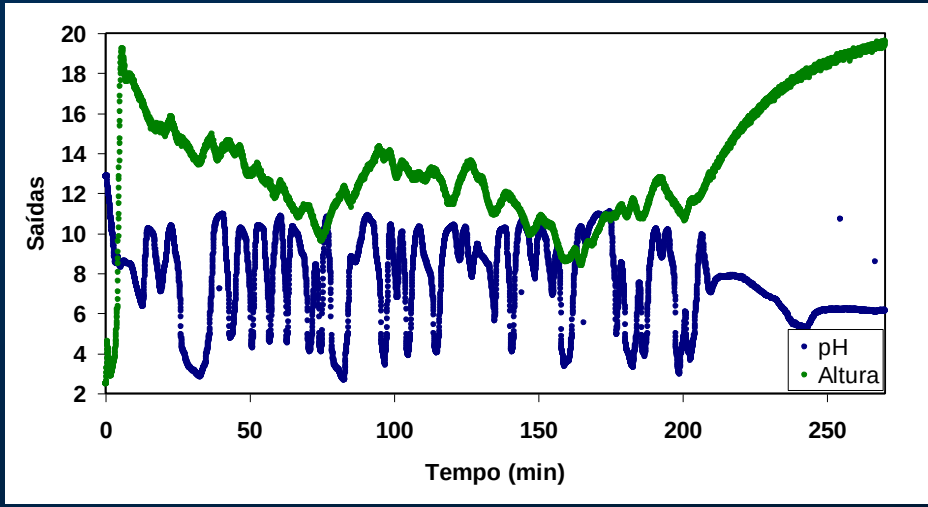






Sistema Experimental: Problemas





CONCLUSÕES

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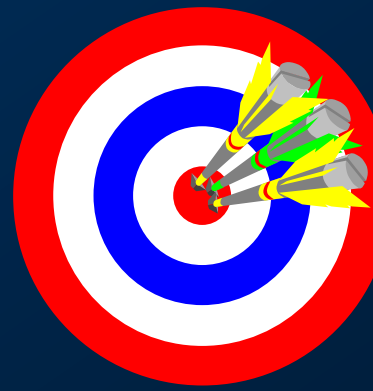
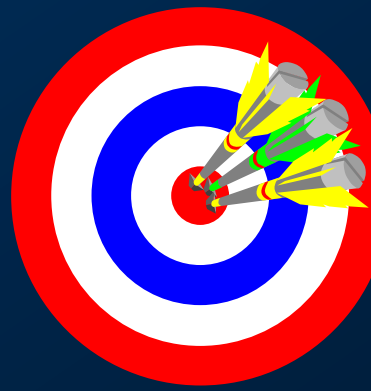


- Proposto algoritmos de RMPC x Objetivos:
  - As saídas dos sistemas mostram-se satisfatórias;
  - As entradas dos sistemas mostram-se adequadas;
  - RMPC_MMR e LMI_RMPCR com desempenho similar a MPC e RMPC clássicos em resultados de simulação e experimental;
  - A planta experimental foi modelada e seus parâmetros identificados → Representativo do processo real;
  - As respostas experimentais para RSSLMPC foram satisfatórias;

CONCLUSÕES



- Proposto algoritmos de RMPC x Objetivos:
  - As repostas teóricas e experimentais do IHMPC
    - Satisfatórias.
  - RMPC_MMR → Satisfatório: teórico e experimental!
  - LMI_RMPCR → Teórico: parcialmente satisfatório;
  - Problemas na Planta...

Sugestões para Trabalhos Futuros:

- Desenvolvimentos Teóricos-computacionais:
  - Esforço computacional;
  - Estudos de análise e desempenho robustos (conceituais);

- Desenvolvimento Experimental:
  - Identificação de modelos de incerteza;
  - Uso de controladores empíricos

"...DEUS ESCOLHE O MELHOR DOS MUNDOS DENTRE TODOS AQUELES QUE SE APRESENTAM COMO POSSÍVEIS..."

GOTTFRIED WILHELM LEIBNIZ

(1646-1716)

AGRADECIMENTOS

E

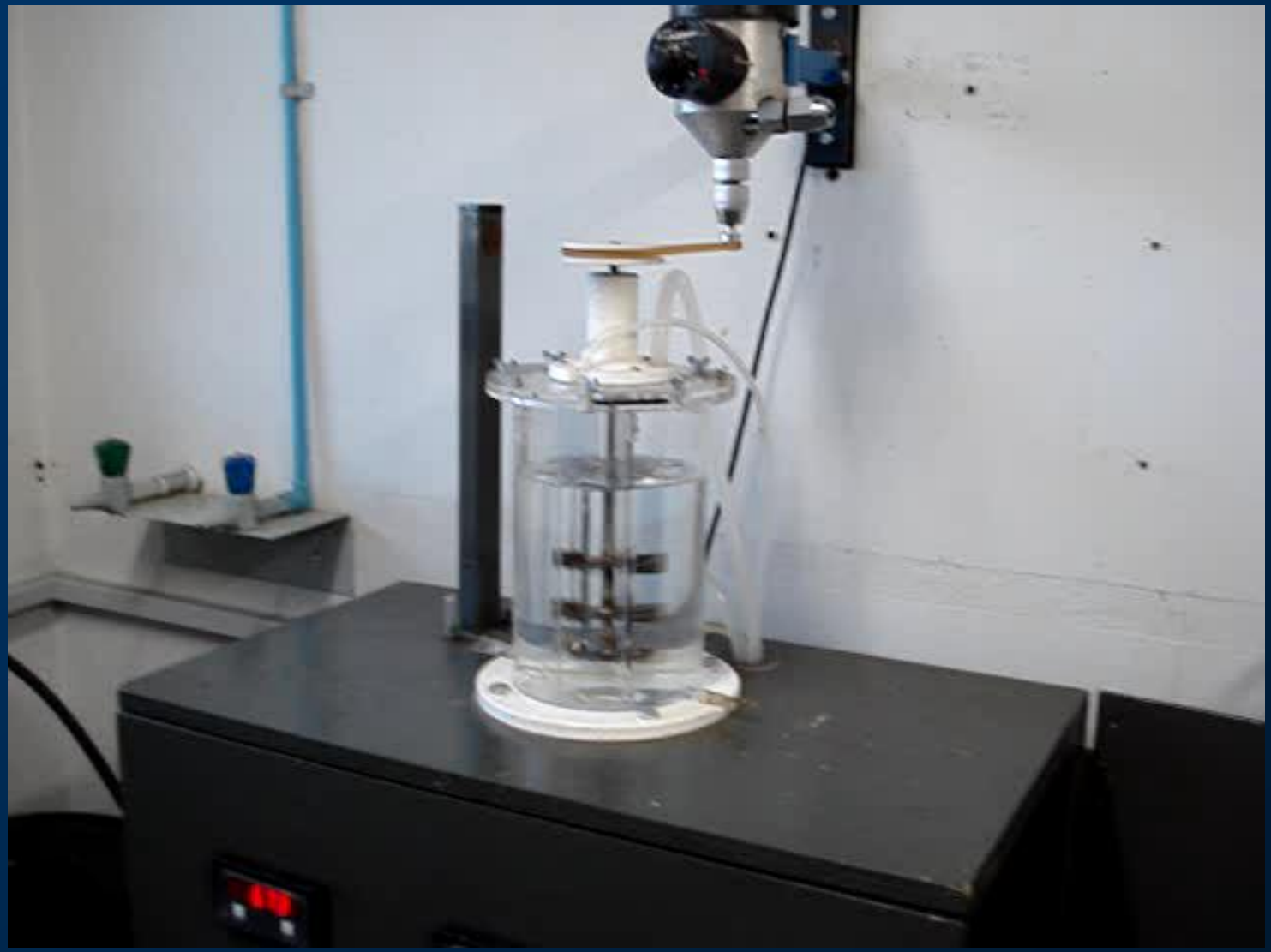
AGRADECIMENTOS

E

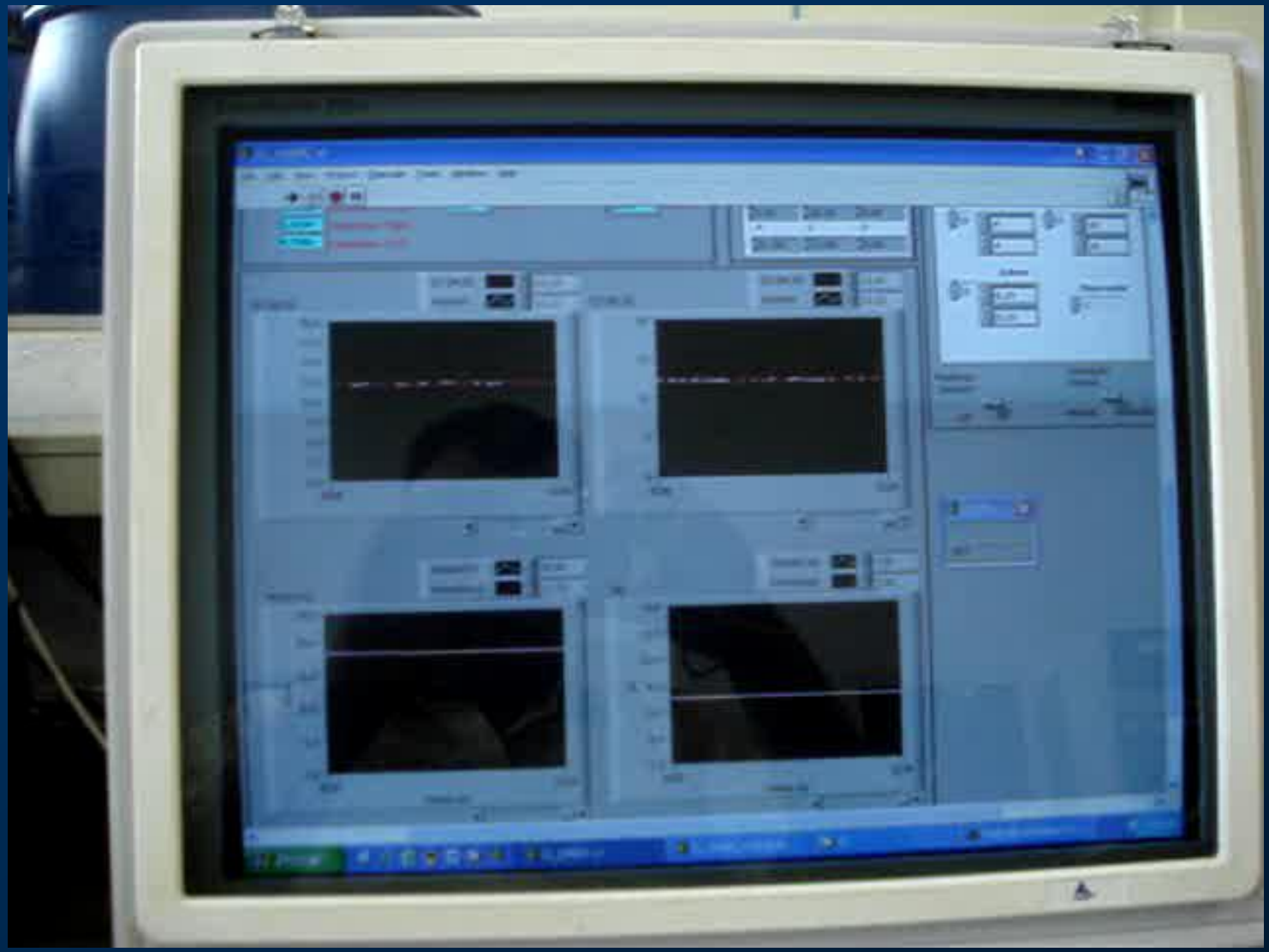
ESCLARECIMENTOS

Sistema Experimental: Vídeos

Sistema Experimental: Vídeos



Sistema Experimental: Vídeos



Sistema Experimental: Vídeos

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Referências principais:

- Motivações:
  - Muske e Rawlings (1993)
  - Bemporad (1999)
  - Mayne et al (2001)
  - Morari (1999)
  - Qin e Badgwell (1996)
  - Skogestad e Postlethwaite (1997)
- RMPC_MMR
  - Lee e Cooley (2000) – RMPC_MMTV e RMPC_MM
  - Badgwell (1997)
- LMI_RMPCR
  - Kothare et al. (1996)
  - Löfberg (2003)